PROSPECT THEORY AND THE INVESTMENT HORIZON

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Prospect Theory (PT) and Constant Relative Risk Aversion (CRRA) have clear-cut and very different implications for the optimal asset allocation as a function of the investment horizon. While CRRA preferences imply that the optimal allocation is independent of the horizon, we show that PT implies a dramatic and discontinuous "jump" in the optimal allocation to stocks as the horizon increases. We experimentally test these predictions. We find rather strong support for CRRA, but very little support for PT. These findings provide some justification for economic models employing CRRA preferences, and suggest that the equity premium puzzle is yet to be solved.

Keywords: *Prospect Theory, constant relative risk aversion, asset allocation, investment horizon, equity premium puzzle.*

JEL Classifications: D81, C91.

1. Introduction

The optimal asset allocation between stocks and bonds, and its dependence on the investment horizon, is one of the most central issues in financial economics.[1] It is a question of great practical importance for investors saving for retirement. This paper theoretically analyzes and then experimentally investigates asset allocation choices corresponding to different investment horizons. We focus on the two main competing preference paradigms: expected utility with Constant Relative Risk Aversion (CRRA), which is widely employed in the economic literature, and Prospect Theory (PT), which has been a dominating paradigm in the last few decades. We first theoretically analyze the changes in the optimal asset allocation as a function of the investment horizon for CRRA and for PT preferences, for general rate of return distributions. PT is analyzed with four alternatives suggested in the literature as the reference point. Based on this theoretical analysis we then experimentally study subjects' asset allocation choices as a function of the investment horizon in a controlled setting, where the two competing preference models have very clear and very different theoretical predictions.

It is well-known that CRRA preferences imply that the asset allocation should be independent of the investment horizon, when returns are i.i.d. and portfolio revisions are allowed after each period (Mossin 1968, Merton and Samuelson 1974). Quite surprisingly, in this study we show that even if revisions are not allowed, which is the setup of our experiment, there is virtually no change in the optimal asset allocation as the horizon increases.

[1] Kelly 1956, Latané 1959, and Markowitz 1976 argue that as the investment horizon grows indefinitely, a stock portfolio, which has a higher geometric mean than a bond (or mixed) portfolio, will almost surely yield a higher terminal wealth than any other alternative portfolio. This supports the idea of stocks for the long run. In contrast, Samuelson 1971 and Merton and Samuelson 1974 show that for CRRA preferences (with portfolio revisions after every period) the optimal investment proportion in stocks is constant, regardless of the length of the investment horizon. Levy (2019) bridges between these views by deriving the optimal asset allocation for CRRA investors under the assumption of log-normal return distributions, and showing that the empirical parameters imply that stocks are preferred to bonds for all investors with RRA<4. For general return distributions, the concept of Almost Stochastic Dominance has been employed to argue for stocks for the long run for *almost* all preferences (Leshno and Levy 2002, Levy 2015), however, this argument has been challenged (Levy 2009).

In contrast, PT preferences imply very different behavior. The PT value function weighs losses more heavily than gains, as captured by the property of loss aversion. Given a stock with a positive mean return, the probability of a loss typically decreases as the investment horizon increases. Thus, as the horizon increases, the stock becomes more attractive. This has led Benartzi and Thaler (1995) to suggest that myopic loss aversion, i.e. PT loss aversion combined with a short evaluation period, can explain the equity premium puzzle. This prediction of PT, that the asset allocation to stocks should increase with the investment horizon is well-known, and has been experimentally tested by Gneezy and Potters (1997) and Thaler, Tversky, Kahneman, and Schwartz (1997). In this study we theoretically confirm that the PT optimal allocation to the stock indeed grows with the investment horizon, as suggested by previous studies. However, perhaps counterintuitively, we show that the optimal allocation to stocks does not grow gradually. Instead, as the horizon increases, the optimal allocation "jumps" dramatically and discontinuously to 100% in the stock (or more, if borrowing at the risk-free rate is possible). This property does not depend on the return distribution, the specifics of the PT value function parameters, or on the use of the cumulative prospect theory (CPT) decision weights. It is also robust to different modelling assumptions about the reference point, which can be either at the current wealth, the future value of wealth if invested in the risk-free asset, the future value of wealth if invested in the stock (namely, a stochastic reference point), or the expected value of wealth given the subject's choice, as suggested by Kőszegi and Rabin (2006). We analytically derive and intuitively explain this peculiar behavior. With realistic PT and return parameters the jump to an optimal weight of 100% in stocks occurs at very short horizons of 2-3 years.

These very specific and very distinct predictions of the two competing models allow us to make clear-cut inference about preferences from the choices in the experiment. We find rather strong support for CRRA preferences, with 44% of the subjects being consistent with these preferences. In contrast, we find that only 6% of the subjects are consistent with PT preferences (for a very wide range of parameters, and alternative reference points).

Several previous studies have examined the implications of PT to asset allocation choice. Berkelaar, Kouwenberg, and Post (2004) derive the optimal dynamic asset allocation for a PT investor in a continuous-time setting, where prices follow an Ito process and markets are complete. They find, as we do, that the optimal allocation to stocks increases with the investment horizon. Gomes (2005) examines the trading volume implications of PT and finds a positive non-linear relation between trading volume and stock return volatility. He also derives the optimal asset allocation between a stock and a risk-free asset and finds that unless the stock's expected return exceeds a certain threshold (which depends on the investor's loss-aversion parameter) the investor's optimal proportion in the stock is zero, also consistent with our results.[2] The two papers most closely related to the present study are the experimental studies by Gneezy and Potters (1997) (GP), and by Thaler, Tversky, Kahneman, and Schwartz (1997) (TTKS), who experimentally investigate the influence of the investment horizon on the *average* asset allocation, aggregated across all subjects. They find that when subjects are presented with longer horizons, on average they tend to increase their investment proportions in the stock, consistent with the predictions of myopic loss aversion. We find similar results regarding the average proportion. However, when we examine choices on the *individual* level, and compare them with the predictions of PT, we find very little support for PT. The two main contributions of the present study relative to the existing literature are: 1) the theoretical identification of a discontinuous jump in the optimal asset allocation of PT investors as the investment horizon increases (or as the stock's return distribution gradually

[2] These papers investigate the asset allocation between stocks (or a stock index) and the risk-free asset. For a discussion of the optimal diversification *across stocks* for a PT investor see Levy and Levy (2003) and Levy, De Giorgi and Hens (2012).

improves), and 2) the experimental investigation of this prediction at the individual level. Section 6 discusses the relation of our results to the existing literature in detail.

In the next section we derive the theoretically optimal asset allocation choices implied by PT. Section 3 derives the theoretical predictions for CRRA preferences (with no rebalancing). Section 4 describes the experiment, and Section 5 provides the results. Section 6 reconciles our results with the existing literature. Section 7 concludes with a discussion of the implications of the findings for economic theory and for investors and pension funds.

2. Optimal Asset Allocation for Prospect Theory investors

The Prospect Theory value function is given by:

$$V(x) \quad = \quad \begin{cases} -\lambda(-x)^{\beta} & for\ x \le 0 \\ x^{\alpha} & for\ x \ge 0 \end{cases}, \qquad (1)$$

where x is the change of wealth relative to the reference point, the exponents $0 < \alpha \le 1$ and $0 < \beta \le 1$ imply risk-aversion for gains and risk-seeking for losses, and the constant $1 < \lambda$ is known as the loss-aversion parameter. Tversky and Kahneman (1992) experimentally estimate $\alpha = \beta = 0.88, \quad \lambda = 2.25$.

In what follows, we assume $\alpha = \beta$. There are two justifications for this assumption. First, experimental studies that estimate these parameters find that they are either exactly equal (Tversky and Kahneman 1992, Camerer and Ho 1994, and Wu and Gonzalez 1996), or that they are very close to each other (Abdellaoui 2000, and Abdellaoui et. al. 2005). Second, $\alpha \ne \beta$ theoretically implies accepting some fair symmetric gambles, in contradiction to the fundamental notion of loss aversion (Levy 2010)[3].

[3] For example, suppose that $\lambda = 2, \quad \alpha = 0.8,$ and $\beta = 0.6$. Consider a fair symmetric gamble that yields either $+a$ or $-a$ with equal probabilities, and a PT investor with reference point at the current wealth. The

Regarding the reference point relative to which gains or losses are measured, there are four main variations suggested in the literature:

1) the future value of the current wealth if invested at the risk-free rate,
2) the current wealth,
3) the expected value of future wealth, given the investor's choice, and
4) the future value of the current wealth if invested in the stock.

All of these alternatives imply a dramatic jump in the optimal allocation to the stock as the horizon increases. Below we analyze alternative 1) in detail. The corresponding analysis for alternatives 2)-4) are provided in Appendix A.

Case 1: Reference point at the Future Value of Wealth Invested at the Risk-Free Rate

If the investment proportion in the stock is *w*, then the terminal wealth is:

$$\tilde{W}_T = W_0\left[w\tilde{R} + (1-w)R_f\right], \tag{2}$$

where W_0 is the initial wealth, $\tilde{R}$ is the stochastic return on the stock (1+rate of return), and R_f is the return on the risk-free asset (1+riskfree interest rate). When the reference point is the future value of the current wealth if invested at the risk-free rate, $W_0 R_f$, the change in wealth relative to the reference is thus:

$$\tilde{x} \equiv \tilde{W}_T - W_0 R_f = W_0\left[w\tilde{R} + (1-w)R_f - R_f\right] = W_0 w\left[\tilde{R} - R_f\right]. \tag{3}$$

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expected value of this gamble is $\frac{1}{2}\left[a^{0.8} - 2a^{0.6}\right]$, and it is positive for a>32. Thus, a PT investor with the above parameters will accept this fair symmetric gamble for any value of a>32. Similarly, for any other $\alpha \neq \beta$ there are some fair symmetric gambles that will be accepted (for any λ). Only $\alpha = \beta$ ensures that all fair symmetric gambles are rejected. See Levy (2010) for more detail.

Denoting the return in *excess* of the risk-free rate by $\tilde{r} \equiv \tilde{R} - R_f$, the PT expected value is given by:

$$EV(w) = W_0^{\alpha} w^{\alpha} \left[-\lambda \int_{-\infty}^{0} f(r)(-r)^{\alpha} dr \quad + \int_{0}^{\infty} f(r)\, r^{\alpha} dr \right], \tag{4}$$

where $f(r)$ is the probability density function of excess returns. This implies two corner solutions for the optimal asset allocation, depending on the sign of the brackets in eq.(4): if the sign is positive the optimal investment weight in the risky asset is $w^* = 1$, and if the sign is negative we have $w^* = 0$.[4] Namely, the optimal asset allocation is:

$$w^* = \begin{cases} 0 & if \;\; \lambda > \dfrac{\int_{0}^{\infty} f(r)\, r^{\alpha} dr}{\int_{-\infty}^{0} f(r)(-r)^{\alpha} dr} \equiv A \\ \\ 1 & if \;\; \lambda < \dfrac{\int_{0}^{\infty} f(r)\, r^{\alpha} dr}{\int_{-\infty}^{0} f(r)(-r)^{\alpha} dr} \equiv A \end{cases} . \tag{5}$$

The above equation implies that the optimal asset allocation for a PT investor is a corner solution: either zero or one.

We shall see below that as the horizon increases, the optimal allocation to the stock "jumps" from 0 to 1 (unless it is already 1 for the 1-period horizon). To see this, note that if the stock's expected return is higher than the return on the risk-free asset, as the horizon increases, the

[4] Recall that $1 \geq w \geq 0$, as it is assumed that no borrowing or short-selling are allowed. If borrowing is allowed, and the square brackets in eq.(4) are positive, this implies that the investor should lever his position in the stock up to the maximal possible limit. Levy (2010) employs this property to translate the empirical equity premium in different countries to estimates of the loss aversion parameter.

probability that stock yields a higher return than the risk-free asset converges to 1.[5] Thus, as the horizon increases $\int_{-\infty}^{0} f(r)dr$ approaches 0, and as $(-r)^{\alpha}$ is bounded (the rate of return on a stock can't be lower than -100%), the term $\int_{-\infty}^{0} f(r)(-r)^{\alpha}\,dr$ also converges to 0. Hence, when the horizon increases, the term A in eq.(5) increases to infinity, and the optimal allocation to the stock becomes 1 for any finite value of λ. This conforms to the profound intuition of Benartzi and Thaler (1995, 1999): when the investment horizon increases the probability of a loss becomes smaller, and stocks become more attractive relative to the risk-free asset. The contribution of the present analysis is to show that this shift to stocks is not gradual: it occurs as a dramatic and discontinuous jump from an allocation of 0% to the stock to an allocation of 100%. Moreover, as we shall see below, one does not need a very long horizon to obtain this "jump" – for typical parameters it occurs at an investment horizon of 2-3 years.

The above analysis employs the objective probabilities. The results are qualitatively the same if one employs the PT (or CPT) decision weights instead. In this case, one should replace the objective p.d.f $f(r)$ in equations (4) and (5) with the PT or CPT decision-weight p.d.f., $f^{*}(r)$. This does not affect the result of a jump from 0 to 1 in the optimal proportion in the stock as the horizon increases (Appendix B shows this for the return distributions employed in our experiment).

We would like to emphasize that while the focus of the present study is the relationship between the optimal asset allocation and the investment horizon, the dramatic jump in the optimal allocation for a PT investor occurs when any of the return or preference parameters are varied. Namely, eq.(5) implies a dramatic jump from 0 to 1 (or vice versa) when the distribution of excess

[5] Indeed, Bali et. al. (2009) empirically find that the probability of the stock return being higher than the bond return is about 66% for monthly returns, and it gradually increases up to 97%-99% for a five-year horizon.

returns, $f(r)$ changes. This can happen as a result of an increase in the investment horizon, which is our main focus, but can also be a result of a change in the stock return distribution due to changing economic conditions, or to a change in the risk-free interest rate. For example, Figure 1 shows the optimal asset allocation as a function of the risk-free rate, r_f , for a PT investor with the Tversky and Kahneman (1992) parameters $\alpha = \beta = 0.88$, $\lambda = 2.25$, and the empirical stock return distribution, estimated by the annual returns on the S&P500 index during 1997-2016. The optimal allocation is computed by eq.(5). For comparison, the figure also shows the optimal asset allocation for an investor with a logarithmic utility function, found numerically. The figure shows that while the optimal allocation to the stock decreases gradually with r_f for the investor with log preference (recall that as no borrowing or shot-selling are allowed, the allocation is bounded between 0 and 1), the PT investor switches dramatically from an allocation of 1 to the stock to 0, at critical value of $r_f = 0.035$.

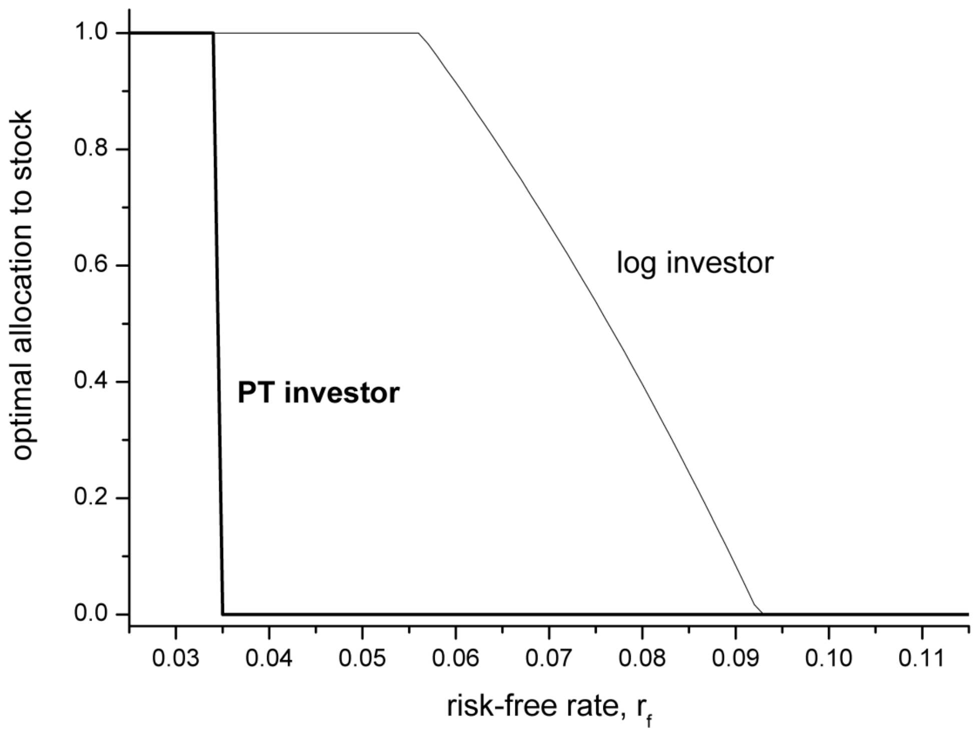

Figure 1: The optimal asset allocation between the S&P500 index and a risk-free asset as a function of the risk-free interest rate r_f. The empirical S&P500 annual returns during 1997-2016 are employed. The bold line shows the optimal allocation to the stock for a PT investor with the Tversky and Kahneman (1992) parameters $\alpha = \beta = 0.88, \quad \lambda = 2.25$, given by eq.(5). The optimal allocation to the stock drops dramatically from 100% to 0% once the risk-free rate reaches 0.035. In contrast, for an investor with a logarithmic utility function (thin line) the optimal allocation to the stock decreases gradually with the risk-free rate (note that the allocation is bounded between 0% and 100%, as no borrowing or short-selling are allowed).

Let us now employ the above general analysis to the specific distributions employed in our experiment. In the experiment subjects are asked to allocate their investment between a risky stock and a risk-free bond. There are three different tasks, where the return distributions are different in each task. Table I provides the three tasks (the detailed experimental procedure is discussed in Section 4). In Task 1, $\tilde{R}$ can take two values, 0.9 or 1.3, with equal probabilities, and the risk-free rate is $R_f = 1.05$. These values correspond to excess returns on the stock of $r = 0.9 - 1.05 = -0.15$ and $r = 1.3 - 1.05 = 0.25$, respectively. Thus, the expected value in Task 1 is given by:[6]

$$EV(w) = \frac{1}{2}(-\lambda)(W_0 w \cdot 0.15)^{\alpha} + \frac{1}{2}(W_0 w \cdot 0.25)^{\alpha} = \frac{1}{2} W_0^{\alpha} w^{\alpha} \left[-\lambda 0.15^{\alpha} + 0.25^{\alpha} \right]. \quad (6)$$

This implies that there are two corner solutions for the optimal asset allocation, w^*:

$w^*=0$ if $-\lambda 0.15^{\alpha} + 0.25^{\alpha} < 0$, and:

$w^*=1$ if $-\lambda 0.15^{\alpha} + 0.25^{\alpha} > 0$.

The threshold point that determines whether the optimal allocation in the stock will be 0 or 100% depends on the value function parameters α and λ. Namely, the optimal investment proportion in the stock, w^*, is given by:

$$w^* = \begin{cases} 0 & if\ \lambda > \frac{0.25^{\alpha}}{0.15^{\alpha}} \\ 1 & if\ \lambda < \frac{0.25^{\alpha}}{0.15^{\alpha}} \end{cases}. \quad (7)$$

[6] The objective probabilities are employed here. Applying CPT decision weights yields the same qualitative results – see the tables in Appendix B.

Table I

The three tasks in the experiment are given below. In each task the subject is asked to allocate his investment between the stock and the risk-free bond. The detailed experimental procedure is provided in Section 4.

Task 1:

If you invest 100% in the stock

Outcome	*Rate of Return*	*Probability*
$90,000	-10%	50%
$130,000	30%	50%

If you invest 100% in the risk-free bond

Outcome	*Rate of Return*	*Probability*
$105,000	5%	100%

Task 2:

If you invest 100% in the stock

Outcome	*Rate of Return*	*Probability*
$81,000	-19%	25%
$117,000	17%	50%
$169,000	69%	25%

If you invest 100% in the risk-free bond

Outcome	*Rate of Return*	*Probability*
$110,250	10.25%	100%

Task 3:

If you invest 100% in the stock

Outcome	*Rate of Return*	*Probability*
$72,900	-27.1%	12.5%
$105,300	5.3%	37.5%
$152,100	52.1%	37.5%
$219,700	119.7%	12.5%

If you invest 100% in the risk-free bond

Outcome	*Rate of Return*	*Probability*
$115,762	15.76%	100%

Panel A of Table II shows the optimal investment proportion in the stock in Task 1 for different combinations of α and λ. For example, for the Tversky and Kahneman (1992) parameters $\alpha = 0.88, \quad \lambda = 2.25$ we have $\lambda = 2.25 > \frac{0.25^{0.88}}{0.15^{0.88}} = 1.56$, and therefore the optimal proportion in the stock is 0. In contrast, for the combination $\alpha = 0.88 \quad \lambda = 1.50$, the optimal proportion is 1 (see the shaded cells in Panel A of Table II).

Similarly, in Task 2 the *EV* is given by:

$$EV(w) = W_0^{\alpha} w^{\alpha}\left[-\frac{1}{4}\lambda 0.2925^{\alpha} + \frac{1}{2}0.0675^{\alpha} + \frac{1}{4}0.5875^{\alpha}\right], \qquad (8)$$

and again we have two corner solutions. The optimal investment proportion in the stock in Task 2 is given by:

$$w^* = \begin{cases} 0 & \text{if } \lambda > \dfrac{\frac{1}{4}0.5875^{\alpha} + \frac{1}{2}0.0675^{\alpha}}{\frac{1}{4}0.2925^{\alpha}} \\ 1 & \text{if } \lambda < \dfrac{\frac{1}{4}0.5875^{\alpha} + \frac{1}{2}0.0675^{\alpha}}{\frac{1}{4}0.2925^{\alpha}} \end{cases}, \qquad (9)$$

(which is a special case of eq.(5) with the return parameters of Task 2). Panel B of Table II provides the optimal investment proportion in the stock in Task 2 for various combinations of α and λ. If we take, for example, the Tversky and Kahneman (1992) parameters $\alpha = 0.88, \quad \lambda = 2.25$ we have:

$\lambda = 2.25 < \dfrac{\frac{1}{4}0.5875^{0.88} + \frac{1}{2}0.0675^{0.88}}{\frac{1}{4}0.2925^{0.88}} = 2.40$, and hence the optimal investment proportion in the stock is 1, i.e. 100%. Thus, a PT investor with the Tversky and Kahneman parameters $\alpha = 0.88 \quad \lambda = 2.25$ is expected to switch from a proportion of 0% in the stock in Task 1, to 100% in the stock in Task 2. Panel C of Table II provides the optimal asset allocation in Task 3, determined by eq.(5) and the return distributions of Task 3. Again, we see the same pattern of two corner solutions.

A comparison of the three panels of Table II reveals that for most combinations of α and λ, a "jump" from 0% in the stock to 100% in the stock occurs as the horizon increases (as one moves either from Task 1 to Task 2, or form Task 2 to Task 3). Thus, with the annual return distribution employed in our experiment, for PT investors, increasing the horizon from 1 year to 3 years implies a jump from 0% in the stock to 100% in the stock (unless the optimal allocation to the stock is already 100% at the 1-year horizon).

Employing Cumulative Prospect Theory (CPT) decision weights instead of the objective probabilities yield similar results. The table in Appendix B shows the optimal asset allocation in Tasks 1-3 when the CPT decision weights are employed ($\gamma = 0.61, \delta = 0.69$, see Tversky and Kahneman 1992; see Bruhin, Fehr-Duda, and Epper 2010 for recent evidence supporting CPT decision weighting).

The alternative cases of a reference point at the current wealth, the expected future wealth given the investor's choice, and the future wealth if invested in the stock yield a similar jump in the optimal asset allocation as the horizon increases. The jump is not necessarily from 0 to 1 in these cases, but rather, from some positive value (e.g. 0.3) to 1. These cases are analyzed in Appendix A.

Table II

Optimal Investment Proportion in the Stock for PT Preferences with Reference Point at the Future Value of Current Wealth ($W_0 R_f$). Three of the highlighted cells correspond to the Tversky and Kahneman (1992) parameters of $\alpha = \beta = 0.88 \quad \lambda = 2.25$ (and one cell in Panel A also to the case $\alpha = \beta = 0.88 \quad \lambda = 1.50$).

A: Task 1

Lambda\alpha	1.00	0.98	0.96	0.94	0.92	0.90	0.88	0.86	0.84
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

B: Task 2

Lambda\alpha	1.00	0.98	0.96	0.94	0.92	0.90	0.88	0.86	0.84
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.75	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

C: Task 3

Lambda\alpha	1.00	0.98	0.96	0.94	0.92	0.90	0.88	0.86	0.84
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.75	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00
2.75	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
3.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

3. Optimal Asset Allocation for CRRA Preferences

It is well-known that for CRRA preferences the optimal proportion is independent of the horizon when *portfolio revisions are allowed after each period* (Mossin 1968, Merton and Samuelson 1974). This is not the case here, because in our setup portfolio revisions are not possible. This is the same setup as in Gneezy and Potters (1997) and in Thaler, Tversky, Kahneman, and Schwartz (1997), and the logic of this setup is explained in Section 4. We show below that despite the fact that revisions are not possible, the optimal asset allocation is *almost* independent of the horizon for CRRA investors. Thus, quite surprisingly, not allowing for portfolio revisions does not change the optimal diversification strategy much, as almost the same optimal diversification is obtained with and without revisions.

The CRRA utility function is given by: $U(W) = \frac{W^{1-\alpha}}{1-\alpha}$, where *W* denotes wealth, and α is the coefficient of relative risk aversion. The optimal investment proportion in the stock in each of the three tasks, for various values of α, is given in Table III. These optimal proportions are calculated numerically by solving for the proportion that maximizes expected utility. Namely, if the investment proportion in the stock is denoted by *w*, the terminal wealth is given by: $\tilde{W} = W_0\left[w\tilde{R} + (1-w)R_f\right]$. For each value of α and each task, we numerically find the optimal proportion w^* that maximizes the expected utility $EU\left[\tilde{W}(w)\right]$, subject to the constraint $0 \leq w \leq 1$, i.e. no borrowing or short-selling. Note that the optimal asset allocation is independent of the initial wealth, W_0.

It is well known that for a given stock return distribution and for a given riskless interest rate, the investment proportion in the stock decreases with an increase in the degree of risk aversion. The result that is more relevant to our study, and that is not obvious, is that for a given value of α, the optimal proportion is almost identical across all three tasks, i.e. it is almost invariant to the investment horizon[7]. For example, if we take the typical case of $\alpha = 2$, we see that the optimal investment proportion in the stock is 68.9% in Task 1, 69.0% in Task 2, and 69.2% in Task 3. This is a very slight increase in the optimal proportion, certainly below the resolution that can be detected experimentally (less than 3% of the subjects answered the questionnaire in resolution of fractions of %). Also, the slight influence of the horizon on the optimal proportion is not the same for all values of α: for $\alpha \leq 2.5$ the proportion slightly increases with the horizon, while for $\alpha \geq 3$ it slightly decreases (see Table III).

Table III

The optimal allocation to the stock in the three tasks for CRRA investors. Note that even though the optimal proportion is not exactly identical across the three tasks, because there are no portfolio revisions, they are almost identical.

Risk aversion	Optimal Investment Proportion in the Stock, *w**		
parameter α	Task 1	Task 2	Task 3
0.5	1.000	1.000	1.000
1.0	1.000	1.000	1.000
1.5	0.924	0.925	0.926
2.0	0.689	0.690	0.692
2.5	0.548	0.549	0.550
3.0	0.456	0.455	0.455
3.5	0.390	0.389	0.388
4.0	0.340	0.339	0.338
4.5	0.302	0.301	0.300
5.0	0.271	0.270	0.269
5.5	0.246	0.245	0.244
6.0	0.226	0.224	0.223
6.5	0.208	0.207	0.206
7.0	0.193	0.192	0.191
7.5	0.180	0.179	0.178
8.0	0.169	0.168	0.166

[7] We assume, like in classic economic models of choices under uncertainty, that the risk aversion parameter at the time of the experiment of each subject remains constant and is independent of the distribution of return the subject faces (see footnote 10).

The explanation for the fact that the optimal proportion is almost constant, even when revisions are not possible, is as follows. First, note that the return distributions in Task 2 are just the distributions obtained by investing in Task 1 for two periods, assuming i.i.d. (and the returns in Task 3 are the 3-period returns). Suppose, for example, that an investor chooses a proportion of $w = 0.5$ in Task 2 (corresponding to the 2-period horizon), and suppose that the stock yields a rate of return of 69%, corresponding to a rate of return of 30% on the stock in both periods. In this case, the rate of return on the investor's portfolio is: $0.5 \cdot 69\% + 0.5 \cdot 10.25\% \; = 39.625\%$. In contrast, suppose that the investor invests $w = 0.5$ in the stock for 1 period, as in Task 1, and that after 1 period he rebalances his portfolio and again invests $w = 0.5$ in the stock for the second period. In this case, his rate of return in the first period is $0.5 \cdot 30\% + 0.5 \cdot 5\% \; = 17.5\%$, and his rate of return in the second period is again 17.5% (again, the stock return is assumed to be 30% in both periods). Thus, after two periods the portfolio grows by $1.175 \cdot 1.175 \; = 1.3806$, i.e. the rate of return on the portfolio after two periods is 38.06%. This is a little lower than the return of 39.625% obtained with no rebalancing.[8] Table IV provides a comparison the entire rate of return distributions with and without revisions for the case of $w = 0.5$. As the table reveals, the return distributions are not exactly identical, but they are very close. We should note that the case of $w = 0.5$ depicted in Table IV is the one where the difference between the two distributions, with and without portfolio rebalancing, is the greatest[9]. As the return distributions are very close, this implies that the optimal asset allocation is very similar whether portfolio revisions are possible or not.

[8]This small difference is due to the fact that after the first period the investor who rebalances sells some of the stock to maintain $w = 0.5$, and thus misses some of the gain from the increase in the stock price in the second period.

[9] In general, if the total return on the stock is R_1 in period 1, and R_2 in period 2, and the total risk-free return is R_f every period, the portfolio total 2-period return is $wR_1R_2 + (1-w)R_f^2$ if no revisions are made, and it is $(wR_1 + (1-w)R_f)(wR_2 + (1-w)R_f)$ if the portfolio is revised after the first period. The difference between these two expressions is: $w(1-w)(R_f(R_1 + R_2) - R_1R_2 - R_f^2)$. This expression is maximal (in absolute value)

In general, for CRRA preferences we have:

- When revisions are allowed, the optimal asset allocation of the investor is independent of the investment horizon (Mossin 1968, Merton and Samuelson 1974).
- For a given asset allocation *w*, the distributions of terminal wealth are almost the same with and without revisions

$$\Downarrow$$

- The optimal asset allocation is almost the same with and without revisions

$$\Downarrow$$

- The optimal asset allocation is almost independent of the investment horizon even when revisions are not possible.

Table IV

The portfolio rate of return distribution for a 2-period horizon, with and without revision of the portfolio weights after the first period. The case shown is for *w*=0.5, which is the case where the difference between the two distributions is maximal (see footnote 16).

probability	Portfolio return without revisions (*w*=0.5)	Portfolio return with revisions (*w*=0.5)
¼	-4.375%	-4.938%
½	13.625%	14.563%
¼	39.625%	38.063%

Indeed, Table III reveals that the optimal investment proportion in the stock with no revisions is almost constant across the three tasks. This result is quite general and does not hinge on the specific distributions given in our experiment: it holds for different return distributions, and for horizons much longer than the 3-period horizon employed in our experiment. For example, if one considers asset allocation between the S&P500 index and the 3-month T-bill, employing the

for *w*=0.5. Obviously, for the cases $w=0$ and $w=1$ the distributions are exactly identical, as no portfolio revisions are made.

empirical annual returns and assuming a 10-year investment horizon, the CRRA optimal asset allocation is very similar with and without revisions: for $\alpha = 2$ the optimal allocation to stocks is 91.1% when annual revisions are allowed, and 91.4% without revisions; for $\alpha = 4$ the optimal allocation to stocks is 47.6% when annual revisions are allowed, and 46.1% without revisions. The figure in Appendix C shows the optimal asset allocation with and without revisions for alphas in the range 0.5-7.5, and provides more detail. As most empirical and experimental evidence suggest that the risk aversion parameter is around 1-2 (see Mehra and Prescott 1985, and references within), the figure reveals that the optimal weight in the risky asset with and without revisions is almost identical. Thus, the optimal asset allocation of a CRRA investor is almost the same whether he invests for 1 year or for 10 years, even if portfolio revisions are not allowed.

In the context of our experiment, this result yields a clear prediction for the asset allocation behavior predicted for CRRA individuals: the investment proportion in the stock should be constant (or almost constant) across all three tasks.

4. The Experiment

The experiment is composed of three main choice tasks. In each task there is a risk-free asset and a risky stock, with a given return distribution. In each task the subject is asked to allocate his investment between these two assets. The instructions that appear before each one of the three tasks are:

Suppose that you have decided to invest $100,000 either in a stock or in a risk-free bond or in any combination of these two assets. The possible outcomes at the end of the investment period are as given below. Please write the percentage of the $100,000 you choose to invest in each asset, where the sum of the two investment proportions should add up to 100%.

Investment proportion in stock: ______ *Investment proportion in risk-free bond:* ______

The three tasks are given in Table I. Note that Task 2 and Task 3 are in fact the 2-period and 3-period versions of Task 1, under the assumption of i.i.d. returns. In other words, the stock's return distribution in Task 2 is the return distribution obtained by investing in the stock given in Task 1 for two periods. Similarly, the risk-free rate in Task 2 is the 2-period return of the risk-free asset of Task 1. In the same manner, Task 3 is the 3-period version of Task 1. These three tasks allow us to investigate asset allocation choices, and their dependence on the horizon. As in Gneezy and Potters (1997) and in Thaler, Tversky, Kahneman, and Schwartz (1997), the tasks are not framed in terms of a 1-period, 2-period and 3-period choices, but rather as three different stand-alone tasks. Specifically, in Tasks 2 and 3 the subjects have no opportunity to revise their allocations after each period. This is important, because as Benartzi and Thaler (1999) have shown, subjects are strongly affected by the information presented to them: if the actual investment horizon is 3 periods, but the information provided to the subjects is for the 1-period returns, subjects tend to treat the investment as a 1-period investment. Thus, to obtain preferences for a 3-period horizon, the 3-period return distributions should be presented, which is the setting of our experiment, as well as in the Gneezy and Potters and the Thaler, Tversky, Kahneman, and Schwartz experiments. In addition, the cash flows of the various choices are obtained immediately, hence subjects do not face anxiety corresponding to immediate and far-away risks.[10] As detailed in Sections 3 and 4, CRRA and PT preferences have very specific and very distinct predictions in this setup.

The experiment is conducted with large hypothetical gains and losses (the maximum gain in Task 3 is $119,700 and the maximum loss is $27,100). This has obvious pros and cons. On the one hand, one may argue that subjects are not sufficiently incentivized when payoffs are hypothetical. On the other hand, this setting allows us to employ large potential gains and losses, which may be perceived by subjects as more realistic. In addition, this may make subjects perceive their decisions

[10] This is important, because there is evidence that the date at which the cash flow is obtained may create anxiety, implying that the risk aversion parameter of the same subject may increase if the same uncertainty is faced shortly (say, next year) relative to the same risk occurring in the distance future, say, ten years from now. For an interesting analyses of this issue see, Van Binsbergen, Brandt, and Koijen (2012), and Eisenbach and Schmalz (2016). In our experiment there is no difference in timing across the three choices: they are all framed as choices with immediate cashflows. Hence, potential timing effects are neutralized.

as more substantial, relative to the case where the outcomes are a few dollars, even if these are translated to actual payoffs for the subject. Most importantly, we wish to create a setup which resembles (under obvious limitations) the subjects' total portfolio asset allocation choice, rather than resembling a small-scale bet. As Rabin (2000) argues, individuals may behave very differently when large stakes are involved, compared to situations with small or moderate stakes.

In order to examine whether the subjects pay attention and consider their choices carefully, we also included a control task, which is the last task of the experiment. In this control task, given in Appendix D, the subject is asked to choose between two alternative investments, where one investment dominates the other by First-order Stochastic Dominance (FSD). Thus, any rational individual (any expected utility maximizer as well as any cumulative PT expected value maximizer) should choose the dominating investment. Indeed, we find that 90% of the subjects chose the dominating investment. This indicates that the vast majority of subjects did pay attention and understood the instructions. We report only results for those subjects who answered the control task correctly. However, as only 10% of the subjects violated FSD, the results do not change much if we include all subjects in the analysis.

The experiment was conducted with a hard-copy questionnaire filled out by pen. Written instructions were given in the questionnaire, as well as provided verbally. There was no show-up fee. The original questionnaire composed of the four tasks discussed above is provided in the online appendix.

Subjects

The subjects are professional investors, and economics/business students. We have four groups of subjects, from three different countries:

Group 1: 57 undergraduate students from China (Harbin Institute of Technology, HIT), average age 24.4, 34% male.

Group 2: 17 master students from Hong Kong (Hong Kong Baptist University), average age 28.2, 67% male.

Group 3: 61 master students (MBA and economics) from Israel (Hebrew University), average age 29.5, 75% male.

Group 4: 68 professional investors from Israel (Harel Mutual Fund Company), average age 37.2, 81% male.

There are a total of 203 subjects. Of these, 184 (90%) answered the control task (with FSD dominance, see Appendix D) correctly. Another two subjects failed to complete the entire questionnaire. Thus, we are left with 182 subjects who completed the questionnaire and answered the control task correctly. The choices across the groups of subjects from various cultures and with different experience in the capital market are quite similar, which increases the reliability of our results. Below we report the results combined across all four groups. Appendix E provides the results for each subject group separately. The individual level data, including group affiliation is provided in the online appendix.

5. Experimental Results

While our focus is the diversification choices at the individual level, the average investment proportions in the risky asset are important, as they are relevant for asset pricing. We find the following average investment proportions in the stock in the three tasks (across all 182 subjects):

$$w_1 = 53.7\% \quad w_2 = 59.0\% \quad w_3 = 58.1\%,$$

where the sub-indices refer to the task number. The average proportion increases somewhat from Task 1 to Task 2, and slightly decreases from Task 2 to Task 3. While the increase in the average proportion from Task 1 to Task 2 is statistically significant (matched-pair t-value 2.44), the decrease from Task 2 to Task 3 is not (t-value -0.12).[11] In general, the average proportions do not change much across tasks. These average proportions *may* be consistent with most subjects having CRRA preferences, where the small variation in the proportions across tasks is due to a small group of subjects with preferences different than CRRA. However, these average proportions *may* also be consistent with many other possible models, including a scenario where some subjects have PT preferences. Thus, in order to reach more definitive conclusions about preferences one cannot rely only on the average proportions, and we therefore turn to analyze the results at the individual level, which is our main experimental focus.

Table V

Asset allocation patterns observed in the experiment. *w* is the investment proportion in the stock, and the sub-index refers to the task number. Patterns 1 and 2 correspond to CRRA and approximate CRRA preferences, respectively. Patterns 3-6 correspond to PT and approximate PT preferences, for a wide range of value function parameters and for different assumptions regarding the reference point.

Pattern	Preference	Condition on proportions in the three tasks	Number of subjects	(%)
1	Exact CRRA	$w_1 = w_2 = w_3$	59	32.4
2	Approximate CRRA	$w_{\max} - w_{\min} < 0.1$	80	44.0
3	PT with reference point either at the future value of wealth or at the expected value of wealth	$w_1 = 0,\ w_2 = 0,\ w_3 = 0$	4	2.2
4	PT with reference point at future value of wealth	$w_1 = 0,\ w_2 = 0,\ w_3 = 1$	0	0
5	PT with reference point at future value of wealth	$w_1 = 0,\ w_2 = 1,\ w_3 = 1$	2	1.1
6	PT with reference point at current wealth	$0.35 > w_1 > 0.25$ $0.40 > w_2 > 0.30$ or $w_2 = 1$ $0.40 > w_3 > 0.30$ or $w_3 = 1$	5	2.7

[11] It is interesting to note that Thaler, Tversky, Kahneman, and Schwartz (1997) observe exactly the same pattern for the average proportions: a significant increase in the average proportion allocated to the stock from the short horizon to the medium horizon, and a slight decrease from the medium horizon to the long horizon. The results are compared in more detail in the next section.

The analysis in Section 3 shows that CRRA preferences predict investment proportions that are practically constant across all tasks, i.e. $w_1 = w_2 = w_3$, regardless of the risk aversion parameter, namely for all CRRA individuals. In contrast, PT preference predicts a jump of the investment proportion in the stock to 100%, either in Task 2 or in Task 3, depending on the preference parameters. Table V summarizes the choice patterns predicted by the different models, and the number of subjects with investment allocation choices conforming to each pattern.

Pattern 1, where the investment proportions are exactly equal across all three tasks, is consistent with CRRA preferences. While there are slight differences in the theoretically optimal proportions across tasks (because there are no revisions, see Table III), these differences are in the order of 0.1%. As almost none of the subjects provided their answers with this degree of detail (more than 97% of subjects answered in whole percentage numbers), we consider this pattern consistent with CRRA. As reported in the table, 59 subjects (32.4%) follow this choice pattern. For each of these subjects we calculated the relative risk aversion coefficient, α, that corresponds to his/her choice. We find an average α value of 1.96, which is similar to values previously reported in the literature (see Mehra and Prescott 1985, and references within).[12] The standard deviation of α is 2.54, indicating a large degree of heterogeneity, even within CRRA subjects.

25 of the 59 subjects following pattern 1 chose $w_1 = w_2 = w_3 = 1$, i.e. they chose to invest 100% in the stock in all three tasks. While this pattern is consistent with CRRA preferences with $\alpha \leq 1$ (see Table III), it could, in principle, be also considered consistent with PT, with $\lambda \leq 1.5$ (see Tables II, AI, and AII). However, we do not consider this choice pattern as supporting PT for two reasons. First, Estimates reported in the literature of the loss aversion coefficient, λ, exceed

[12] This may be an overestimation of α, because subjects who chose to invest 100% in the stock may have low α's and actually wanted to invest more than 100% in the stock (by borrowing at the risk-free rate), which was not possible in our experiment.

1.75, and typically also exceed 2.[13] While we should allow for heterogeneity in λ across subjects, we would expect to have $\lambda > 1.5$ for most PT investors. Thus, according to PT most choices should be according to patterns 3-6 (see below), and only a small minority should be $w_1 = w_2 = w_3 = 1$. Observing only a few choices of patterns 3-6, implies that we are observing *only* the subjects at the left tail of the λ distribution, but not the subjects at the bulk of the distribution with $\lambda > 1.5$. This does not seem reasonable. Second, while a small value of λ is technically consistent with PT, this is in a somewhat degenerate sense, because $\lambda \approx 1$ actually implies almost linear preferences.

The second pattern (see pattern 2 in Table V) allows for some small variation in the proportions across tasks, possibly due to "noise", or bounded rationality errors. We consider all patterns where the maximum difference between the three proportions is smaller than 10% (i.e. $w_{\max} - w_{\min} < 0.1$), as "approximately consistent" with CRRA preferences. We find that 80 subjects (44%) fall into this category. Note that the 59 subjects in pattern 1 are a subgroup of the 80 subjects in pattern 2, i.e. when we allow for small variations in the allocations, 21 subjects are added to the 59 subjects in pattern 1.

We turn now to the choice patterns predicted by PT. We do not confine ourselves to the specific PT parameters suggested by Kahneman and Tversky (1992), but rather, we consider a wide range of possible parameters α, and all possible values of λ larger than 1.75 (see footnote 13).

For $\lambda > 1.75$ PT with a reference point at the future value of wealth predicts $w_1 = 0$ in Task 1, see Panel A of Table II. The investment proportion in the stock then jumps to 1 at either Tasks 2 or 3, or remains at 0, depending on the exact values of λ and α (see Panels B and C of Table II). Pattern 3 (with weights 0, 0, 0), pattern 4 (with weights 0, 0, 1), and pattern 5 (with weights 0,

[13] Kahneman et. al. (1990), and Tversky and Kahneman (1991) estimate that λ is in excess of 2. Tversky and Kahneman (1992) estimate λ at 2.25, and Pennings and Smidts (2003) estimate it as 1.8.

1, 1) of Table VI correspond to these three cases. Pattern 3 is predicted for all PT investors with a reference point at the future value of wealth. In aggregate, only 6 subjects (3.3%) follow one of these three patterns. Thus, even if we allow for a wide range of PT parameters, there is very little support for PT with a reference point at the future value of wealth.

When the reference point is at the current wealth, and $\lambda > 1.75$, w_1 is between 0.30 to 0.33 (see Panel A of Table A1), w_2 is either 0.35 or 1, and w_3 is either 0.37 or 1. In order to allow for some noise, or bounded-rationality errors (as we allow for approximate CRRA in pattern 2), we allow for deviations from these values and define pattern 6 as: $0.35 > w_1 > 0.25$, $0.40 > w_2 > 0.30$ or $w_2 = 1$, and $0.40 > w_3 > 0.30$ or $w_3 = 1$. Notice that PT subjects with a reference point at current wealth should follow this pattern whether they employ the objective probabilities or the CPT decision weights (see the table in Appendix B). Only 5 subjects (2.7%) follow pattern 6. We should note that some cases of pattern 6 may also be consistent (and are a special case of) patterns 1 and 2. For example, a subject who chooses $w_1 = w_2 = w_3 = 0.35$ is approximately consistent with PT (because he is included in pattern 6), but is also perfectly consistent with CRRA. Thus, there is some overlap between the choice patterns, and such a subject would be included in patterns 1, 2, and 6. Of the 5 subjects following pattern 6, four are also consistent with approximate CRRA, i.e. they do not exhibit any "jump". Thus, even when we allow for a wide range of PT parameters we find very weak support for PT: in total, at most 6% of the subjects are consistent with PT. We say "at most" because some of the subjects classified as PT subjects also fall in the CRRA category.

The other subjects who do not fall into any of the categories in Table V, display a variety of choice patterns. 38 subjects (21%) monotonically increase their investment proportion in the stock with the horizon, i.e. $w_1 < w_2 < w_3$ (but not as predicted by PT). For 20 subjects (11%) the investment proportion monotonically decreases: $w_1 > w_2 > w_3$. 29 subjects display non-monotonic proportions: for 18 subjects we have $w_1 < w_2 > w_3$, and for 11 subjects $w_1 > w_2 < w_3$.[14]

To summarize, the most dominant choice pattern is pattern 1, where all proportions are exactly identical. 32.4% of the subjects are exactly consistent with CRRA, and 44% are "approximately consistent" with CRRA. At most 6% of the subjects are approximately consistent with PT.

6. Relation to the Existing Literature

We find evidence supporting CRRA preferences for a relatively large proportion of the subjects. This finding is consistent with previous empirical and experimental findings: Arrow (1971) concludes that the relative risk aversion is almost constant, implying CRRA. Friend and Blume (1975) find support for CRRA based on survey data regarding households' asset allocations. Gordon, Paradis, and Rorke (1972), Kroll, Levy, and Rapoport (1988), and Levy (1994) find experimental evidence in support of CRRA.

We find very weak support for PT preferences: at most 6% of the subjects are consistent with PT, and about half of them are also consistent with CRRA (4 of the 5 subjects following pattern 6). This may seem surprising, in light of the vast experimental support for PT. How can this result be reconciled with the literature? We believe that the most plausible explanation is that individuals behave differently when faced with small or modest bets (as in most experimental studies supporting PT) and when faced with large stakes, as in the present experimental setup. The importance of the scale of the amount at stake on choices is best summarized by Rabin (2000 p. 1286) who writes:

[14] There are a few subjects who do not fall into any of these categories, for example subjects with $w_1 = w_2 > w_3$, $w_1 > w_2 = w_3$, etc.

> *Expected utility theory may well be a useful model of the taste for very large scale insurance. Despite its usefulness, however, there are reasons why it is important for economists to recognize how miscalibrated expected utility is as an explanation of modest-scale risk aversion.*

Another possible explanation is that much of the experimental support for PT comes from experiments with either positive or negative prospects, rather than with mixed prospects. Indeed, Levy and Levy (2002) find support for PT when employing prospects in the positive domain or in the negative domain separately, but find that most subjects contradict PT when mixed prospects are employed. Needless to say, most real-life decisions involve both potential gains and potential losses, and this is also the framework of the present study. It is also possible that PT is approximately consistent with aggregate results, while it does not provide a good description of choice at the individual level. This is what Diecidue, Levy and van de Ven (2015) find.

The two papers most closely related to the present study are those by Gneezy and Potters (1997) (GP), and by Thaler, Tversky, Kahneman, and Schwartz (1997) (TTKS). Both of these studies experimentally investigate the influence of the investment horizon on the *average* asset allocation. They find that when subjects are presented with longer horizons, on average they tend to increase their investment proportions in the stock. This is consistent with myopic loss aversion – when individuals with PT preferences observe short-horizon returns they are confronted with a large probability of a loss, and therefore they tend to avoid the stock; as the horizon increases, the probability of a loss decreases, and individuals tend to increase their investment proportions in the stock (Benartzi and Thaler 1995, 1999). At the aggregate level, this is similar to what we observe here: the average proportion in the stock increases from 53.7% in Task 1 to 59% in Task 2 (and then slightly decreases at Task 3). However, as discussed above, these aggregate results may also be consistent with most subjects having CRRA preferences, and the relatively small increase in the average investment proportion being driven by a (potentially small) group of investors with non-CRRA preferences. TTKS report a large heterogeneity in subjects' behavior (see their Figure 1, p.

655). This implies that individual behavior may be very different than the averages reported. Thus, only by looking at decisions at the individual level we can reach more definitive conclusions regarding preferences. In the present study we can predict not only that the optimal investment allocation to the stock should increase with the horizon, but also the exact way that it is predicted to increase. Thus, the two main distinctions between GP and TTKS and the current study is that we analyze choices at the *individual level*, and that we have very clear predictions about the *exact asset allocation values* implied by PT.

One key difference between GP and TTKS is that GP investigate asset allocation between a risky stock and a risk-free bond (as in the present study), while TTKS study the asset allocation between a risky stock and a risky bond (which is less volatile than the stock, but still involves risk). This has an important implication for the optimal asset allocation of a PT investor: when the bond is risk-free, a dramatic jump in the asset allocation is theoretically predicted, as discussed in Section 2. Indeed, in the GP setup PT also implies a jump from 0% to 100% in the stock.[15] When the bond is risky (as in TTKS, and also in Benartzi and Thaler 1995) the optimal asset allocation may change

[15] In the Gneezy and Potters (1997) experiment subjects have to decide what part of an endowment they would like to allocate to a stock that yields a rate of return of -100% with probability 1/3, and a rate of return of 250% with probability 2/3 in the one-period setting. If the proportion allocated to the stock is *w*, the expected value is: $EV = -\frac{2}{3}\lambda(wW_0)^{\alpha} + \frac{1}{3}(2.5wW_0)^{\alpha} = \frac{1}{3}W_0{}^{\alpha}w^{\alpha}\left[-2\lambda + 2.5^{\alpha}\right]$. Thus, the optimal solution is *w*=0 for $\lambda > \frac{2.5^{\alpha}}{2}$, and *w*=1 for $\lambda < \frac{2.5^{\alpha}}{2}$. Similarly, in the three-period setting of their experiment, some algebraic manipulation reveals that the optimal solution is *w*=0 for $\lambda > \frac{7.5^{\alpha} + 6\cdot 4^{\alpha} + 12\cdot 0.5^{\alpha}}{8\cdot 3^{\alpha}}$, and *w*=1 for $\lambda < \frac{7.5^{\alpha} + 6\cdot 4^{\alpha} + 12\cdot 0.5^{\alpha}}{8\cdot 3^{\alpha}}$ (the returns are additive in the GP setting). For $\alpha = 1$, these threshold values are λ =1.25 in the one-period setting and λ =1.56 in the three period setting. Thus, subjects with $\alpha = 1$ and $1.56 > \lambda > 1.25$ should switch from 0 investment in the stock in the one-period setting to 100% in the stock in the three-period setting. Subjects with $\lambda > 1.56$ should not allocate any money to the stock, in either setting.

more gradually. This is an advantage of the GP setup, and the setup employed here: it yields very clear and testable theoretical predictions.

It is interesting to note that TTKS find that: *"…subjects made exactly the same choices (on average) when they were allocating assets one period at a time or when they were committing themselves for 400 trials…"* (p. 656). It is well-known that this behavior is consistent with CRRA preferences when revisions are allowed. In Section 3 we show that even when revisions are not allowed, as in the final decision in the TTKS setup, and as in our experiment, CRRA implies almost the same behavior. Thus, this finding by TTKS actually seems to lend support for CRRA.

7. Conclusions

This study employs the setting of investments for different horizons to theoretically analyze the predictions of two main contending preferences: Constant Relative Risk Aversion (CRRA) preferences, and Prospect Theory (PT) preferences (with and without decision-weights, and with various alternative reference points). We then experimentally investigate whether subjects' choices conform to these predictions. The special feature of this setup is that it yields very clear predictions for the individual-level behavior of CRRA and PT preferences. We show that for CRRA preferences, regardless of the risk aversion parameter, the asset allocation should remain virtually constant at different horizons, even when revisions are not allowed. In contrast, we show that for PT preferences the optimal allocation to the risky stock should "jump" dramatically and discontinuously from about 30% (or 0%, depending on the reference point) to 100% (or even more, if borrowing is possible) as the horizon increases. It is important to note that this jump does not require a very long horizon, it already occurs at shifts from a one-year horizon to a two-year or a three-year horizon. These theoretical results are quite general: they do not depend on the specific return distributions, the specific PT parameters, and whether objective probabilities or CPT decision-weights are employed. It is important to emphasize that the jump in the asset allocation to the risky asset is a fundamental characteristic of PT and it occurs even if the horizon does not change, e.g., if the stock distribution is fixed and the risk-free interest rate increases. Generally, we do not obtain such a jump in the expected utility framework.

Like in Gneezy and Potters (1997) and in Thaler, Tversky, Kahneman, and Schwartz (1997), in our experiment each subject faced three choices, not knowing that they correspond to various horizons (or to a combination of several lotteries). Thus, by the selected choices we can investigate the horizon effect on asset allocation, neutralizing the effect of the timing of cashflows (see, for example, Van Binsbergen, Brandt, and Koijen (2012), and Eisenbach and Schmalz (2016)). We find that 32% of the subjects' choices are exactly consistent with the predictions of CRRA. If we allow for some bounded-rationality errors, this figure increases to 44%. We find that *at most* 6% of the subjects are consistent with PT, even if we allow for some bounded-rationality errors, a wide range of PT parameters, and different assumptions regarding the reference point. This is an upper bound, because about one half of the subjects consistent with PT are also consistent with CRRA.

This very weak support for PT is surprising, given the large number of experimental studies supporting PT. We suspect that this difference may be partly due to the fact that many of these studies investigate the domain of gains and the domain of losses separately, while we employ mixed prospects. More importantly, it is possible that loss aversion plays an important role in setups where gains and potential losses are modest (e.g. a few tens of dollars), but when larger amounts are at stake, such as lifelong savings, CRRA provides a much better model of preferences. In his famous paper, Rabin (2000) shows that CRRA (and expected utility in general) are not consistent with actual behavior "in the small" and "in the large": realistic risk aversion over modest outcomes implies absurd risk aversion over large outcomes. We complement this result by showing that PT is also not consistent with behavior both "in the small" and "in the large": while PT may explain choices over modest outcomes very well, in a setting of lifelong savings with large outcomes it yields very unrealistic predictions. It is perhaps time to develop unifying models of preference that merge these very different behaviors "in the small" and "in the large".

Our findings have several key implications. The first is about the equity premium puzzle (Mehra and Prescott 1985). Benartzi and Thaler (1995) suggest that this puzzle may be solved by myopic loss aversion: PT preferences and a short horizon (or a high evaluation frequency). The lack of experimental support that we find for PT preferences "in the large" casts doubt on the validity of this explanation. We must conclude, like Mehra and Prescott (2003), that the equity premium puzzle is yet to be solved.

The second implication of our results is about the important role of heterogeneity. We find that subjects are very different not only along one dimension (such as the degree of risk aversion), but rather they follow completely different patterns of the asset allocations as a function of the horizon (some monotonically increasing the allocation to the stock with the horizon, some monotonically decreasing it, and some changing it non-monotonically). As Lucas (1976), Kirman (1993), Blackburn and Ukhov (2013), and others have argued, one should be very careful about reaching conclusions about the aggregate behavior of very heterogeneous individuals.

Having said that, we find that CRRA is the preference class that best describes subjects' behavior when large outcomes are at stake. Thus, if one must choose a single preference to model choice "in the large", CRRA seems to be the best alternative, by far.

Finally, when periodic portfolio revisions are possible, CRRA preferences imply that the optimal asset allocation is independent of the investment horizon. We show that this result approximately holds even when revisions are *not* possible. Thus, the "stocks for the long run" argument does not hold for CRRA investors, either with or without revisions. PT may explain the preference for stocks as the horizon increases, but this is not in the spirit of the "stock for the long run" argument, as PT predicts that investors should dramatically switch from a low investment proportion in the risky asset to a proportion of 100% (or more, if possible), once the investment horizon exceeds 2-3 years. This is an unrealistic result which has no empirical or experimental support.

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Appendix A: Optimal Asset Allocation for a PT Investor with Alternative Reference Points

Case 2: Reference point at Current Wealth

In this case we also obtain a discontinuous jump in the optimal investment proportion in the stock. The difference relative to Case 1 (described in the text) is that the jump is not from 0 to 1, but rather from some positive value (about 1/3 for the distributions given in the experiment) to 1. Again, this result is general, and does not depend on the specific return distribution. In what follows we will first provide the general analysis, and then discuss the specific tasks used in the experiment.

As in Case 1, the terminal wealth is given by: $\tilde{W}_T = W_0\left[w\tilde{R} + (1-w)R_f\right]$. However, the change of wealth is calculated relative to current wealth, W_0, i.e.:

$$\tilde{x} \equiv \tilde{W}_T - W_0 = W_0\left[w\tilde{R} + (1-w)R_f - 1\right] = W_0\left[w\left(\tilde{R} - R_f\right) + R_f - 1\right]. \qquad (10)$$

We denote the excess return on the stock by $\tilde{r} \equiv \tilde{R} - R_f$ and $r_f \equiv R_f - 1$. Without loss of generality, we take $W_0 = 1$. Then, we have:

$$\tilde{x} = w\tilde{r} + r_f . \qquad (11)$$

We assume that the expected excess return is positive $E[\tilde{r}] > 0$ and that the interest rate is positive $r_f > 0$. Thus, if *r* is positive, then the change of wealth, *x*, is positive for any $1 \geq w \geq 0$. If *r* is negative, *x* can be either positive or negative, depending on *w*. Let us denote the minimum value of $\tilde{r}$ by $r_{\min}$. In any non-degenerate situation we have $r_{\min} < 0$ (if $r_{\min} > 0$, the minimum return on the stock is higher than the risk-free return, and the stock thus dominates the risk-free asset by First-order Stochastic Dominance). $r_{\min}$ is also bounded from below, because the rate of return on the stock can't be lower than -1 (-100%).

From eq.(11) we conclude that if $w < -\frac{r_f}{r_{\min}}$ then $\tilde{x}$ is positive for any value of $\tilde{r}$, and we have:

$$EV = \int_{r_{\min}}^{\infty} \left(wr + r_f\right)^{\alpha} f(r)\, dr\,. \tag{12}$$

If $w > -\frac{r_f}{r_{\min}}$ then $\tilde{x}$ can be either negative or positive. Specifically, it is negative for $r < -\frac{r_f}{w}$,

and it is positive for $r > -\frac{r_f}{w}$ (see eq.(11)). Thus, in the range $w > -\frac{r_f}{r_{\min}}$ we have:

$$EV = -\lambda \int_{r_{\min}}^{-\frac{r_f}{w}} \left(-\left(wr + r_f\right)\right)^{\alpha} f(r)\, dr \quad + \int_{-\frac{r_f}{w}}^{\infty} \left(wr + r_f\right)^{\alpha} f(r)\, dr\,. \tag{13}$$

The PT expected value given by eqs.(12) and (13) is continuous across the two ranges, but its derivative with respect to *w* is not. Let us analyze each of these ranges separately.

Range 1 $w < -\frac{r_f}{r_{\min}}$ **:**

The first and second derivatives of the *EV* in eq.(12) with respect to the investment proportion, *w*, are: (14)

$$\frac{\partial EV}{\partial w} = \alpha \int_{r_{\min}}^{\infty} \left(wr + r_f\right)^{\alpha-1} r\ f(r)\, dr \quad \text{and} \quad \frac{\partial^2 EV}{\partial w^2} = \alpha(\alpha - 1) \int_{r_{\min}}^{\infty} \left(wr + r_f\right)^{\alpha-2} r^2\ f(r)\, dr\,.$$

The first derivative may generally be either positive or negative, because *r* is negative in the range $[r_{\min}, 0]$ and positive in the range [0, ∞]. The second derivative is negative: notice that the integral in the second derivative is positive (because $wr + r_f > 0$), and the PT risk-aversion in the domain of gains implies $0 < \alpha < 1$. Also, the first derivative at *w*=0 is positive (because the expected value of *r* is positive). Thus, in the range $\left[0, -\frac{r_f}{r_{\min}}\right]$ the maximum value of *EV* is obtained either at some internal point $0 < w < -r_f / r_{\min}$, or at the end point $w = -r_f / r_{\min}$. Notice that the existence of a maximum point in the range $\left[0, -\frac{r_f}{r_{\min}}\right]$ is independent of the distribution $f(r)$, i.e. of the investment horizon, and of the preference parameters (although the exact location of the maximum point does depend on these factors).

When we consider the whole range of the investment weight in the stock, $1 \geq w \geq 0$, the above maximum point may be either the global maximum or a local maximum, depending on the behavior of the *EV* at values of *w* exceeding $-r_f / r_{\min}$: if the *EV* decreases in the range $w > -r_f / r_{\min}$, then the above maximum point is global. However, if the *EV* increases in the range $w > -r_f / r_{\min}$, the global maximum may be in found in this second range. Thus, we turn next to analyze the *EV* in the range $w > -r_f / r_{\min}$, namely, in the general case where the change of wealth, *x*, may be either negative or positive.

Range 2 $w > -\frac{r_f}{r_{\min}}$**:**

In this range, the derivative of the *EV* in eq.(13) with respect to *w* is given by[16]:

[16] Notice that as the limits of the integrals in eq.(13) are also functions of *w*, one should employ the Leibnitz integral rule. In our specific case this does not change the value of the derivative, because the value of the function integrated at the integral limit which depends on *w* is zero. Let us elaborate. The Leibnitz rule is:

$$\frac{\partial}{\partial w}\left(\int_{a(w)}^{b(w)} g(w,r)dr\right) = g(w,b(w))\frac{\partial b(w)}{\partial w} - g(w,a(w))\frac{\partial a(w)}{\partial w} + \int_{a(w)}^{b(w)} \frac{\partial g(w,r)}{\partial w} dr .$$

$$\frac{\partial EV}{\partial w} = -\lambda\alpha \int_{r_{\min}}^{-\frac{r_f}{w}} \left(-\left(wr + r_f\right)\right)^{\alpha-1} (-r) f(r)\, dr \quad + \alpha \int_{-\frac{r_f}{w}}^{\infty} \left(wr + r_f\right)^{\alpha-1} r\, f(r)\, dr\,. \qquad (15)$$

It is convenient to divide the second integral to its positive and negative parts:

(16)

$$\frac{\partial EV}{\partial w} = -\lambda\alpha \int_{r_{\min}}^{-\frac{r_f}{w}} \left(-\left(wr + r_f\right)\right)^{\alpha-1} (-r) f(r)\, dr \quad + \alpha \int_{-\frac{r_f}{w}}^{0} \left(wr + r_f\right)^{\alpha-1} r\, f(r)\, dr + \alpha \int_{0}^{\infty} \left(wr + r_f\right)^{\alpha-1} r\, f(r)\, dr$$

The first two terms above are negative, while the third is positive: in the first term both $-\left(wr + r_f\right)$ and $-r$ are positive and we have a minus sign in front of the integral, hence the first term is negative; in the second and third terms $wr + r_f$ is positive, while r is negative in the second term, but is positive in the third term. The overall sign of the derivative in eq.(16) depends on the relative magnitudes of these three terms.

If we take, for example, the derivative of the first term in eq.(13), we have $g(w,r) \equiv \left(-\left(wr + r_f\right)\right)^{\alpha} f(r)$, and therefore:

$$\frac{\partial}{\partial w}\left(\int_{r_{\min}}^{-\frac{r_f}{w}} \left(-\left(wr + r_f\right)\right)^{\alpha} f(r)\, dr \right) = \left(-\left(w\left(-\frac{r_f}{w}\right) + r_f\right)\right)^{\alpha} f(-\frac{r_f}{w}) \frac{\partial}{\partial w}\left(-\frac{r_f}{w}\right) - \left(-\left(wr_{\min} + r_f\right)\right)^{\alpha} f(r_{\min}) \frac{\partial}{\partial w}\left(r_{\min}\right) +$$

$$\alpha \int_{r_{\min}}^{-\frac{r_f}{w}} \left(-\left(wr + r_f\right)\right)^{\alpha-1} (-r) f(r)\, dr.$$

The first term is equal to 0 because the value of the function g at the upper limit is 0, and the second term is 0 because the lower limit does not depend on w. Thus, we are left with only the third term. Similarly, in the derivative of the second term in eq.(13), the first term is 0 because the upper limit does not depend on w, and the second term is 0 because the value of the function g at the lower limit is 0, and again, we are left with only the third term, which is $\alpha \int_{-\frac{r_f}{w}}^{\infty} \left(wr + r_f\right)^{\alpha-1} r\, f(r)\, dr$.

For short horizons the probability that the excess return *r* is negative is typically substantial. Thus, the first two terms, which are integrals over *r* in the range $[r_{\min}, 0]$ are relatively large, because of the overweighing of the first term by the loss aversion factor λ, and the overall derivative may be negative. Thus, for short horizons the *EV* my decrease in the range $w > -r_f / r_{\min}$, implying that the maximum *EV* is obtained in the range $0 < w \le -r_f / r_{\min}$. However, as the horizon increases, the probability that the excess return is negative decreases to 0, and the first two terms in eq.(16) become smaller, while the third term becomes larger. Thus, as the horizon increases the derivative (16) becomes positive, for any value of $w > -r_f / r_{\min}$, and the optimum investment weight jumps from somewhere in the range $0 < w^* \le -r_f / r_{\min}$ to $w^* = 1$.[17]

[17]Actually, if borrowing at the risk-free rate is possible (which is not the setup of our experiment) and the reference point is at the current wealth, as the horizon increases the optimal investment proportion in the stock becomes infinite. To see this, it is helpful to analyze the derivative in eq.(16) in the case $w >> r_f$. Note that eq.(16) can be rewritten as:

$$\frac{\partial EV}{\partial w} = \alpha w^{\alpha-1}\left[-\lambda \int_{r_{\min}}^{-\frac{r_f}{w}} \left(-\left(r + \frac{r_f}{w} \right) \right)^{\alpha-1} (-r) f(r)\, dr \quad + \int_{-\frac{r_f}{w}}^{0} \left(r + \frac{r_f}{w} \right)^{\alpha-1} r\, f(r)\, dr + \int_{0}^{\infty} \left(r + \frac{r_f}{w} \right)^{\alpha-1} r\, f(r)\, dr \right].$$

When $w >> r_f$, this expression reduces to:

$\frac{\partial EV}{\partial w} \cong \alpha w^{\alpha-1}\left[-\lambda \int_{r_{\min}}^{0} (-r)^{\alpha} f(r)\, dr \quad + \int_{0}^{\infty} r^{\alpha} f(r)\, dr \right]$. By integration over *w*, this implies that for large *w* we have $EV \cong w^{\alpha}\left[-\lambda \int_{r_{\min}}^{0} (-r)^{\alpha} f(r)\, dr \quad + \int_{0}^{\infty} r^{\alpha} f(r)\, dr \right]$, just like in the case of a reference point at the future value of wealth (see eq.(4)). As the horizon increases, the probability of a negative excess return shrinks, and for any λ there comes a horizon where $\lambda < \int_{0}^{\infty} f(r)\, r^{\alpha} dr \Big/ \int_{-\infty}^{0} f(r)(-r)^{\alpha}\, dr$, i.e. the square brackets become positive. This implies that for large *w* the derivative is positive and constant, i.e. it is optimal to borrow at the risk-free rate and invest an infinite amount in the stock. Note that the convergence to the case of a reference point at the future value of wealth holds for $w >> r_f$. For lower values of *w*, the optimal asset allocation is different when the different reference points are employed.

Thus, like in Case 1, with the reference point at the future value of wealth invested in the risk-free asset, we again obtain a jump in the investment weight in the risky asset, albeit not from zero to 1 but from some positive value smaller than 1 to 1. Before discussing the optimal asset allocation in the tasks employed in our experiment, let us first take a short detour to address the following question: From a practical/empirical perspective, how long does the horizon have to be for the jump to occur? After all, if the jump occurs at a horizon of 100 years, this is not of much practical interest. In order to answer this question, we perform the following analysis employing the empirical return distributions. We consider the optimal asset allocation between the S&P500 index and 3-month T-bills. Figure A1 shows the optimal proportion in the index as a function of the investment horizon, for various PT parameters. The T-year return distribution for the stock index is estimated by randomly drawing 50,000 sets of T returns (with replacement) from the set of annual returns on the S&P500 index during 1997-2016. The T-bill rate is taken as the average annual rate over this period (2.14%). For a PT investor with λ=3 and α=0.88 the optimal investment proportion in the stock index is 9% if the horizon is 1 year, and it jumps to 100% if the horizon is 2 years or longer. (For a PT investor with the Tversky and Kahneman (1992) parameters of with λ=2.25 and α=0.88, which is less loss-averse, the optimal proportion in stocks is 100% already at the 1-year horizon, and remains 100% at all horizons). For an investor with λ=3 and α=0.6 the optimal investment proportion in the stock index is 5% if the horizon is 1 year, 13% for 2 years, 27% for 3 years, and it then jumps to 100%. Similarly, other PT preference parameters also yield dramatic jumps in the optimal allocation. Figure A1 reveals that the jump typically occurs at very short horizons of 2-3 years. Even with rather extreme loss aversion, λ=5, and high risk aversion, α=0.6, a PT investor with a horizon longer than 6 years should invest 100% in stocks (and if borrowing is possible, up to the maximal leveraged position possible).

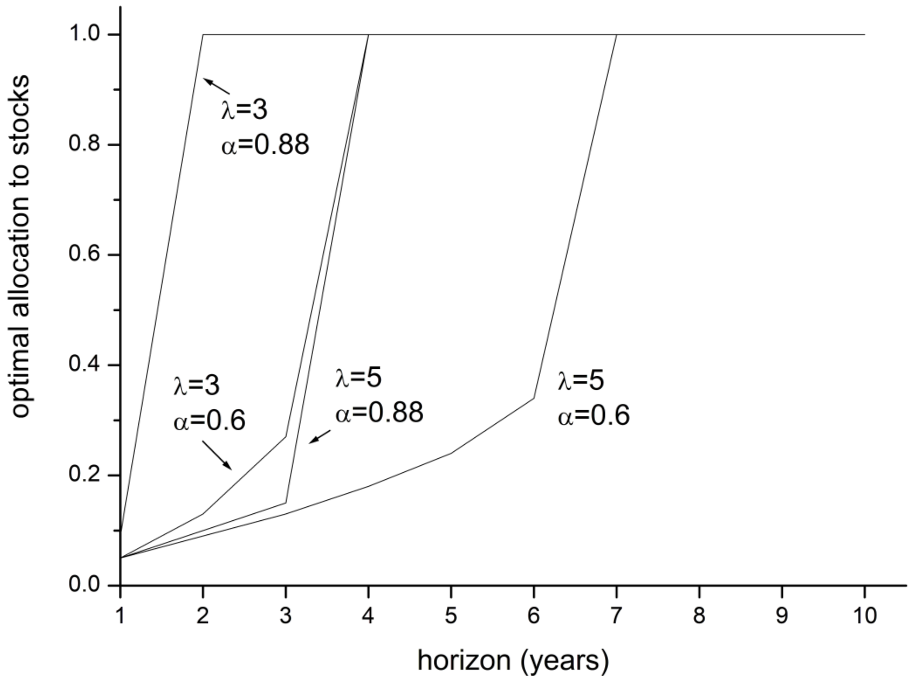

Figure A1: The optimal asset allocation between the S&P500 index and 3-month T-bills for PT investors as a function of the investment horizon. We employ the empirical S&P500 returns during 1997-2016 to construct the return distributions at various horizons. We construct the T-year return distribution by randomly drawing 50,000 sets of T returns (with replacement) from the set of annual returns on the S&P500 index during 1997-2016. The T-bill rate is taken as the average annual rate over this period (2.14%). The optimal allocation to the stock typically grows gradually with the horizon to a value in the range 10%-30%, and then "jumps" dramatically to 100%.

We should note that this dramatic jump in the asset allocation contrasts the common practice of the widely popular life cycle mutual funds. These funds *gradually* decrease the proportion allocated to risky assets every few years, as the savers become older, and the expected horizon shortens. In contrast, as Figure A1 shows, for the case where the reference point is at the current wealth, PT advocates an allocation of 100% in stocks for all but the shortest horizons, and then a sharp decrease to an allocation of only 10%-30% in stocks for very short horizons (or to 0% if the reference point is at the future value of wealth).

Let us now return to the specific prospects employed in our experiment. In Task 1 the stock yields either R=1.3 or R=0.9 with equal probabilities. If R=1.3, we have:

$$x = \left[w(1.3 - 1.05) + 1.05 - 1\right] = \left[0.25 \cdot w + 0.05\right], \tag{17}$$

and if R=0.9 we have:

$$x = \left[w(0.9 - 1.05) + 1.05 - 1\right] = \left[-0.15 \cdot w + 0.05\right]. \tag{18}$$

(Recall that without loss of generality we take the initial wealth to be $W_0 = 1$). While the change of wealth in eq.(17) is positive for any w, the sign of the change of wealth in eq.(18) depends on w: this expression is positive (i.e. it is considered as a gain) if w<1/3, however, if w>1/3, x is negative (i.e. a loss). Thus, we can write the EV in Task 1 as:

$$EV(w) = \begin{cases} \frac{1}{2}\left[(0.25w + 0.05)^{\alpha} + (-0.15w + 0.05)^{\alpha}\right] & if \quad w \le 1/3 \\ \\ \frac{1}{2}\left[(0.25w + 0.05)^{\alpha} - \lambda(0.15w - 0.05)^{\alpha}\right] & if \quad w > 1/3 \end{cases} . \tag{19}$$

To see that this expression implies corner solutions, it is instructive to first concentrate on the simplified case of a piecewise linear value function where $\alpha = 1$ (as discussed by Benartzi and Thaler 1995). In this case, plugging $\alpha = 1$ simplifies eq.(19) to:

$$EV(w) = \begin{cases} \frac{1}{2}\left[0.1 \cdot w + 0.1\right] & if \quad w \le 1/3 \\ \\ \frac{1}{2}\left[(0.25 - 0.15\lambda)w + 0.05(1 + \lambda)\right] & if \quad w > 1/3 \end{cases} . \tag{20}$$

As in the general case, the *EV* is continuous in *w*, but its derivative is not. Figure A2 plots the *EV* in eq.(20) as a function of *w*. The *EV* increases linearly up to the point *w*=1/3. For *w*>1/3 the *EV* either continues to increase or starts to decrease, depending on the sign of the expression 0.25-0.15λ. If λ<0.25/0.15=1.66, this expression is positive and the *EV* continues to increase linearly in the range *w*>1/3, implying that the optimal investment proportion in the stock is 1. If λ>1.66 this expression is negative, implying that the *EV* decreases linearly in the range *w*>1/3, and thus that the maximum *EV* is obtained at *w*=1/3.

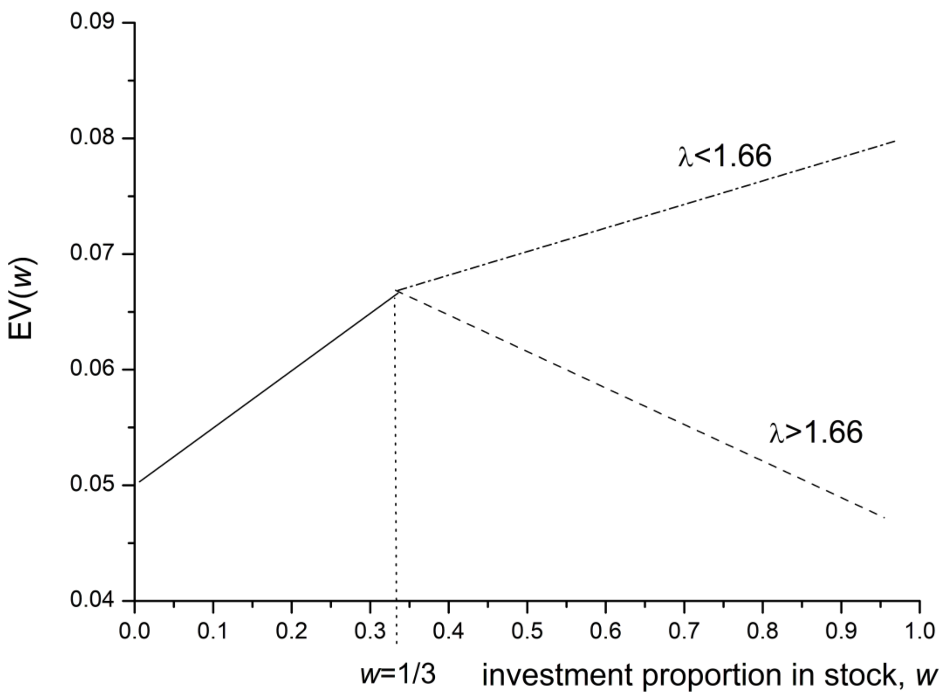

Figure A2: The PT expected value as a function of the investment proportion in the stock in Task 1. The reference point is the current wealth, and a piecewise linear value function is employed ($\alpha = 1$). There are two possible optimal allocations: *w*=1/3 for λ>1.66, and *w*=1 for λ<1.66.

When $\alpha \neq 1$ the results are similar, but as the *EV* is not linear in *w*, the maximum EV may be obtained at values lower than 1/3. The derivative of the *EV* with respect to *w* in the range *w*<1/3 is:

$$\frac{\partial EV}{\partial w} = 0.25\alpha(0.25w + 0.05)^{\alpha-1} - 0.15\alpha(-0.15w + 0.05)^{\alpha-1} \quad , \qquad (21)$$

and the maximum EV in this range is obtained at[18]:

$$w^* = \frac{0.05 \cdot \left(\frac{3}{5}\right)^{\frac{1}{\alpha-1}} - 0.05}{0.15 \cdot \left(\frac{3}{5}\right)^{\frac{1}{\alpha-1}} + 0.25} . \qquad (22)$$

In the range *w*>1/3 the derivative is either positive for all *w*, or negative for all *w*, depending on the values of α and λ.[19] Thus, the optimal proportion is either given by eq.(22), or is 1.

For $\alpha = 0.88$ the optimal investment proportion in the range *w*<1/3 is obtained at:

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[18] Equating the derivative in eq.(21) to zero yields: $\frac{0.05 + 0.25w^*}{0.05 - 0.15w^*} = \left(\frac{0.15}{0.25}\right)^{\frac{1}{\alpha-1}} = \left(\frac{3}{5}\right)^{\frac{1}{\alpha-1}}$. Note that for $\alpha \to 1$ we obtain *w**=1/3 as a special case of eq.(22).

[19] In the range *w*>1/3 the derivative is:
$\frac{\partial EV}{\partial w} = \frac{1}{2}\left[0.25\alpha(0.25w + 0.05)^{\alpha-1} - 0.15\lambda\alpha(0.15w - 0.05)^{\alpha-1}\right]$. Equating this derivative to zero (in search for a maximum point) yields: $w^* = \frac{0.05 \cdot \left(\frac{3}{5}\lambda\right)^{\frac{1}{\alpha-1}} + 0.05}{0.15 \cdot \left(\frac{3}{5}\lambda\right)^{\frac{1}{\alpha-1}} - 0.25}$. A little algebra reveals that this expression is either negative (if $\lambda > \left(\frac{5}{3}\right)^{\alpha}$) or larger than 1 (if $\lambda < \left(\frac{5}{3}\right)^{\alpha}$). Thus, there is no maximum (or minimum) point in the range 1>*w*>1/3. Note that for $w >> 0.05$ the *EV* becomes $EV = \frac{1}{2}w^{\alpha}\left[0.25^{\alpha} - \lambda 0.15^{\alpha}\right]$, and continuous to grow (or decline) indefinitely with *w*.

$$w^* = \frac{0.05 \cdot \left(\frac{3}{5}\right)^{\frac{1}{0.88-1}} - 0.05}{0.15 \cdot \left(\frac{3}{5}\right)^{\frac{1}{0.88-1}} + 0.25} = 0.32\,. \qquad (23)$$

Thus, for $\alpha = 0.88$ there are two possible solutions in Task 1: either 0.32 or 1, depending on the value of λ. Panel A of Table AI provides the optimal asset allocation in Task 1 for various combinations of α and λ, when the reference point is taken as current wealth. Panels B and C provide the corresponding optimal asset allocations in Tasks 2 and 3 (calculated numerically).

Comparing the three panels of Table AI, corresponding to the case of a reference point at current wealth, reveals similar "jump" pattern in the optimal asset allocation as in Case 1, where the reference point is at the future value of wealth. As the horizon increases, the optimal proportion in the stock jumps dramatically from values of about 1/3 to 1. For example, for the Tversky and Kahneman (1992) parameters of $\alpha = 0.88$ $\lambda = 2.25$ the optimal investment proportion in the stock increases from 0.32 in Task 1 to 0.35 in Task 2, and then jumps to 1 in Task 3. The results are similar when CPT decision weights are employed instead of the objective probabilities, as shown in table A2.

To summarize Case 2, with a reference point at current wealth, we prove that in the general case there is a jump in the PT optimal allocation to the risky asset as the horizon increases. The jump is from some value smaller than (or equal to) $-r_f/r_{min}$ for short horizons to 1 (or more, if borrowing is possible) for longer horizons.

Table AI

Optimal Investment Proportion in the Stock in the three tasks for PT Preferences with Reference Point at the Current Wealth (W_0). The highlighted cells correspond to the Tversky and Kahneman (1992) parameters of $\alpha = \beta = 0.88$ $\lambda = 2.25$. Objective probabilities are employed.

A: Task 1

Lambda\alpha	1.00	0.98	0.96	0.94	0.92	0.90	0.88	0.86	0.84
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.30
1.50	1.00	1.00	1.00	0.33	0.33	0.33	0.32	0.31	0.30
1.75	0.33	0.33	0.33	0.33	0.33	0.33	0.32	0.31	0.30
2.00	0.33	0.33	0.33	0.33	0.33	0.33	0.32	0.31	0.30
2.25	0.33	0.33	0.33	0.33	0.33	0.33	0.32	0.31	0.30
2.50	0.33	0.33	0.33	0.33	0.33	0.33	0.32	0.31	0.30
2.75	0.33	0.33	0.33	0.33	0.33	0.33	0.32	0.31	0.30
3.00	0.33	0.33	0.33	0.33	0.33	0.33	0.32	0.31	0.30

B: Task 2

Lambda\alpha	1.00	0.98	0.96	0.94	0.92	0.90	0.88	0.86	0.84
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.75	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2.00	1.00	1.00	1.00	1.00	1.00	1.00	0.35	0.35	0.35
2.25	1.00	1.00	1.00	0.35	0.35	0.35	0.35	0.35	0.35
2.50	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35
2.75	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35
3.00	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35

C: Task 3

Lambda\alpha	1.00	0.98	0.96	0.94	0.92	0.90	0.88	0.86	0.84
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.75	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2.75	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.37

Table AII

Optimal Investment Proportion in the Stock in the three tasks for PT Preferences with Reference Point at the Current Wealth (W_0) and with CPT decision weights, $\gamma = 0.61, \delta = 0.69$.

A: Task 1

Lambda\alpha	1.00	0.98	0.96	0.94	0.92	0.90	0.88	0.86	0.84
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.25	1.00	1.00	1.00	1.00	1.00	1.00	0.31	0.30	0.28
1.50	1.00	0.33	0.33	0.33	0.33	0.32	0.31	0.30	0.28
1.75	0.33	0.33	0.33	0.33	0.33	0.32	0.31	0.30	0.28
2.00	0.33	0.33	0.33	0.33	0.33	0.32	0.31	0.30	0.28
2.25	0.33	0.33	0.33	0.33	0.33	0.32	0.31	0.30	0.28
2.50	0.33	0.33	0.33	0.33	0.33	0.32	0.31	0.30	0.28
2.75	0.33	0.33	0.33	0.33	0.33	0.32	0.31	0.30	0.28
3.00	0.33	0.33	0.33	0.33	0.33	0.32	0.31	0.30	0.28

B: Task 2

Lambda\alpha	1.00	0.98	0.96	0.94	0.92	0.90	0.88	0.86	0.84
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.75	1.00	1.00	1.00	1.00	1.00	1.00	0.35	0.35	0.34
2.00	1.00	1.00	1.00	0.35	0.35	0.35	0.35	0.35	0.34
2.25	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.34
2.50	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.34
2.75	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.34
3.00	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.34

C: Task 3

Lambda\alpha	1.00	0.98	0.96	0.94	0.92	0.90	0.88	0.86	0.84
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.75	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.37
2.50	1.00	1.00	1.00	1.00	1.00	0.37	0.37	0.37	0.37
2.75	1.00	1.00	1.00	0.37	0.37	0.37	0.37	0.37	0.37
3.00	1.00	1.00	0.37	0.37	0.37	0.37	0.37	0.37	0.37

Case 3: Reference point at the Expected Value of Wealth

In this case the reference point depends on the return distributions under consideration, and on the subject's asset allocation choice. If the investor invests a proportion w in the stock, then his terminal wealth is:

$\tilde{W}_T = W_0\left[w\tilde{R} + (1-w)R_f\right]$, and the expected value of terminal wealth is:

$\bar{W}_T = W_0\left[w\bar{R} + (1-w)R_f\right]$. The change of wealth relative to this reference point is:

$$\tilde{x} \equiv \tilde{W}_T - \bar{W}_T = W_0 w[\tilde{R} - \bar{R}]. \tag{24}$$

Denoting the deviation of the stock's return from its average return by $\tilde{z} \equiv \tilde{R} - \bar{R}$, the PT expected value is given by:

$$EV(w) = W_0^{\alpha} w^{\alpha}\left[-\lambda \int_{-\infty}^{0} f(z)(-z)^{\alpha} dz \;+\; \int_{0}^{\infty} f(z) z^{\alpha} dz\right], \tag{25}$$

where $f(z)$ is the probability density function of z. Similar to Case 1, again, this implies a corner solution for the optimal asset allocation, depending on the sign of the square brackets in eq.(25): if the sign is positive the optimal investment weight in the risky asset is $w^* = 1$, and if the sign is negative we have $w^* = 0$. Specifically, the optimal asset allocation is given by:

$$w^* = \begin{cases} 0 & if \;\; \lambda > \dfrac{\int_0^{\infty} f(z) z^{\alpha} dz}{\int_{-\infty}^{0} f(z)(-z)^{\alpha} dz} \equiv B \\ \\ 1 & if \;\; \lambda < \dfrac{\int_0^{\infty} f(z) z^{\alpha} dz}{\int_{-\infty}^{0} f(z)(-z)^{\alpha} dz} \equiv B \end{cases} . \tag{26}$$

Exactly as in Case 1, where the reference point is $W_0 R_f$, here too we find a discontinuous jump in the optimal asset allocation from 0 to 1 as the return distribution changes. Notice that if the return distribution $\tilde{R}$ is symmetric, then the distribution of $\tilde{z}$, the distance of the return from the mean return, is symmetric around 0. In this case, the term B in eq.(26) is equal to 1, and thus the optimal allocation to the stock is 0 for any PT investor with $\lambda \geq 1$. Thus, in Task 1, which has a symmetric return distribution, the optimal allocation to the stock is 0. In Tasks 2 and 3 we also have B<1, and thus, again, the optimal allocation to the stock is 0 for any loss-averse PT investor.[20] Thus, the prediction of PT with a reference point at the expected value of wealth is an allocation of 0 to the stock in all three tasks of our experiment.[21]

Case 4: Stochastic Reference point at the Future Value of Wealth Invested in the Stock

Many funds managers are evaluated relative to a benchmark or index (Sharpe 1992). Some individual investors may also consider their reference point as the alternative of investing in the risky stock. This has led De Giorgi and Post (2011) to suggest a state-dependent stochastic reference point, which is the wealth if invested in the stock. This stochastic reference point also implies a corner solution for the optimal asset allocation, and a discontinuous jump for 0 to 1 in the optimal allocation to the stock, as shown below.

The investor's reference point is his terminal wealth if invested in the stock, i.e. $W_0\tilde{R}$. If the investor invests a proportion w in the stock, then his terminal wealth is:

$\widetilde{W}_T = W_0\left[w\tilde{R} + (1-w)R_f\right]$, and the change of wealth relative to the stochastic reference point is:

[20] For example, in Task 2 we have $\bar{R} = 1.21$,
and $\tilde{z} = (-0.4 \;\; with\, prob.\, 0.25; \; -0.04 \;\; with\, prob.\, 0.5; \;\; 0.48 \;\; with\, prob.\, 0.25;)$. This implies $B = \frac{0.25\cdot 0.48^{\alpha}}{0.5\cdot 0.04^{\alpha} + 0.25\cdot 0.4^{\alpha}}$, or $= \frac{0.48^{\alpha}}{2\cdot 0.04^{\alpha} + 0.4^{\alpha}}$, which is smaller than 1 for any $\alpha < 1$. Similarly, in Task 3 we have $\bar{R} = 1.331$ and $= \frac{\frac{3}{8}0.190^{\alpha} + \frac{1}{8}0.866^{\alpha}}{\frac{3}{8}0.278^{\alpha} + \frac{1}{8}0.602^{\alpha}}$, which is, again, smaller than 1 for any $\alpha < 1$.

[21] The setup with reference point at the expected value of wealth typically implies an optimal allocation of 1 to the stock when the return distribution is negatively skewed. For example, if the rate of return on the stock is -50% with probability 0.1, and 20% with probability 0.9, we have $\bar{R} = 1.13$ and $B = \frac{0.9\cdot 0.07^{\alpha}}{0.1\cdot 0.63^{\alpha}}$. For $\alpha = 0.8$, for example, we have $B = 1.55$, and thus for any PT investor with $\lambda < 1.55$ the optimal investment proportion in the stock is 1.

$$\tilde{x} \equiv \tilde{W}_T - W_0\tilde{R} = W_0(1-w)\left[-\left(\tilde{R} - R_f\right)\right]. \qquad (27)$$

Denoting the stock return in *excess* of the risk-free rate by $\tilde{r} \equiv \tilde{R} - R_f$, we have: $\tilde{x} = W_0(1-w)(-\tilde{r})$, and the PT expected value is given by:

$$EV(w) = W_0^\alpha(1-w)^\alpha\left[-\lambda\int_0^\infty f(r)r^\alpha dr \ + \int_{-\infty}^0 f(r)(-r)^\alpha dr\ \right]. \qquad (28)$$

(Notice that when r is positive x is negative, i.e. this is a loss. This is the reason that the integral over the positive values of r are multiplied by the loss aversion coefficient λ). Similar to the cases analyzed in the text, this again implies a corner solution for the optimal asset allocation, depending on the sign of the square brackets in eq.(28): if the sign is positive the optimal investment weight in the risky asset is $w^* = 0$, and if the sign is negative we have $w^* = 1$. Specifically, the optimal asset allocation is given by:

$$w^* = \begin{cases} 0 & if \ \ \lambda < \dfrac{\int_{-\infty}^0 f(r)(-r)^\alpha dr}{\int_0^\infty f(r)r^\alpha dr} = \dfrac{1}{A} \\ \\ 1 & if \ \ \lambda > \dfrac{\int_{-\infty}^0 f(r)(-r)^\alpha dr}{\int_0^\infty f(r)r^\alpha dr} = \dfrac{1}{A} \end{cases} . \qquad (29)$$

Eq.(29) implies a discontinuous jump in the optimal asset allocation from 0 to 1 as the return distribution changes, just like in Case 1. The intuition for eq.(29) is the following: the investor dislikes deviations from the stock, which is his reference point, so a large λ induces allocation of 1 to the stock. The investor is willing to deviate from holding the stock only if i) his λ is not very large, and ii) the performance of the stock relative to the risk-free asset is not so good, i.e. the fraction is eq.(29) is large. Notice that the fraction in eq.(29) is exactly the inverse of the fraction in eq.(5) given in the text, which we denoted by A. The conditions (5) and (29) are not identical:

while eq.(5) implies that the optimal allocation to the stock is 100% if $\lambda < A$, eq.(29) implies this result for $\lambda > 1/A$. While these are different conditions, they do imply the same qualitative behavior: as the horizon increases, and the probability that the stock yields a higher return than the risk-free asset approaches 1, A increases indefinitely, and we have both $\lambda < A$ and $\lambda > 1/A$, i.e. both cases imply an allocation of 100% to the stock. However, the jump to the stock may occur at different horizons under the two setups with alternative reference points.

Appendix B:

Optimal Investment Proportion in the Stock for PT Preferences with Reference Point at the Future Value of Current Wealth invested at the risk-free asset ($W_0 R_f$) and CPT decision weights, $\gamma = 0.61, \delta = 0.69$.

A: Task 1

Lambda\alpha	1.00	0.98	0.96	0.94	0.92	0.90	0.88	0.86	0.84
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.50	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00
1.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

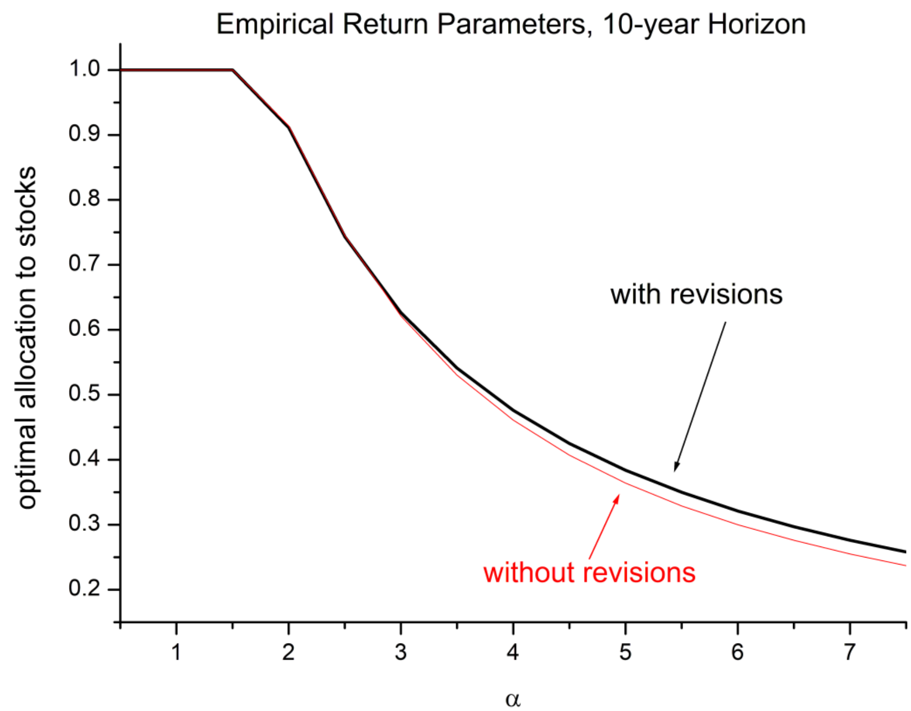
B: Task 2

Lambda\alpha	1.00	0.98	0.96	0.94	0.92	0.90	0.88	0.86	0.84
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.75	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

C: Task 3

Lambda\alpha	1.00	0.98	0.96	0.94	0.92	0.90	0.88	0.86	0.84
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.75	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2.50	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00
2.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Appendix C



The optimal asset allocation between the S&P500 index and 3-month T-bills for CRRA investors with a 10-year investment horizon. The optimal allocation is calculated when the portfolio can be revised annually (bold line), and when no revisions are possible (thin line). We employ the empirical returns during 1997-2016. For the 10-year horizon, the 10-year return distributions are estimated by randomly drawing 50,000 sets of 10 returns (with replacement) from the annual returns on the S&P500 index during 1997-2016. The T-bill rate is taken as the average annual rate over this period (2.14%). The figure shows that the optimal asset allocation is almost the same whether revisions are allowed or not. Thus, a CRRA investor should optimally allocate approximately the same proportion to stocks whether he invests for one year or for ten years, even if portfolio revisions are not possible.

Appendix D: The Control Task

The following task was used as a control, to filter out subjects who did not take the questionnaire seriously, or failed to comprehend the experimental setup:

Suppose that you can invest the $100,000 either in investment F or in investment G (but you cannot diversify between the two investments) with the following end of period values:

Investment F		**Investment G**	
Outcome	*Probability*	*Outcome*	*Probability*
$110,000	.50	$110,000	.25
$120,000	.50	$120,000	.50
		$130,000	.25

Which investment will you choose? __________ (Please write F or G)

Investment G dominates investment F by First-degree Stochastic Dominance (FSD). Thus, we expect all rational subjects to choose G. Indeed, 90% of the subjects chose G.

Appendix E: The Average Allocation to the Stock, by Group and by Task

The average asset allocations in a given task are similar across the four subject groups. The differences between groups are not statistically significant. Specifically, the null hypothesis of homogenous investment choices of the four groups in Task I, Task II and Task III cannot be rejected (p= 51.9%, 45.2% and 7.7% , respectively). When we regress the individual investment proportions in a given task on group dummy variables none of the dummy coefficients are significant.

Group		Tasks			Average across tasks
		Task I	Task II	Task III	
1	China (undergraduate students)	52.4%	55.2%	54.8%	54.1%
2	Hong Kong – (Master students)	46.7%	55.4%	63.4%	55.2%
3	Israel – (Master students)	58.8%	63.8%	66.0%	62.9%
4	Israel – (Professional investors)	52.2%	56.9%	52.8%	54.0%
	Average across groups	53.7%	59.0%	58.1%	56.7%