CLARIFYING MANAGERIAL BIASES USING A PROBABILISTIC FRAMEWORK

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ABSTRACT

A unified probabilistic framework is developed to analyze and compare the impact of the psychological biases of overconfidence, optimism, underconfidence and pessimism on managerial perceptions about the expected value, overall risk, downside risk, value-at-risk (VaR) and expected shortfall (ES) of decision-making variables. The results depict that overconfident and optimistic managers overestimate and underconfident and pessimistic underestimate their expected values. Overall risk is underestimated by the overconfident managers but overestimated by the other three configurations. Downside risk, VaR and ES measures are severely underestimated by the overconfident and optimistic managers and overestimated by the underconfident and pessimistic managers.

Keywords: Downside risk; expected shortfall; probability miscalibration; psychological biases; skewed normal distribution; value-at-risk.

JEL: G4, C46

1. Introduction

Managerial decisions are often out of line with the rational expectations, e.g., Shefrin (2001) and (2005) and Baker et al. (2006). This is generally due to behavioral biases, such as, overconfidence, unrealistic optimism, underconfidence and pessimism, attributed to cognitive shortcuts and emotional errors, e.g., Statman (2017). For example, overconfident managers, including CEOs, generally believe that their firms are undervalued, therefore, tend to hold executive options for longer periods, e.g., Palmon et al., 2008; Palmon and Venezia (2013) and (2015); and Malmendier and Tate (2015). Galasso and Simcoe (2011) found a positive relationship between overconfidence and patents counts in more competitive industries. Moreover, optimistic managers, because of overestimation, often invest in negative net present value projects, see Heaton (2002). In summary, the literature alludes to the behavioral biases of overconfidence and optimism. The similarities, the differences and implications of these configurations are often misinterpreted.

Overconfidence has been explained by hubris and miscalibration, e.g., Oberlechner and Osler (2012). Hubris occurs when persons overestimate their own success or the probability of favourable outcomes, e.g., Roll (1986) and Camerer and Lovallo (1999). Calibration measures the accuracy of probabilities. It is a probabilistic tool in decision making, e.g., Lichtenstein et al. (1982). Often, the perceived probability distribution of overconfident people is characterized as too tight, e.g., Alpert and Raiffa (1982) and Lichtenstein et al. (1982) and that of underconfident people as too loose, e.g., Kyle and Wang (1997). In other words, overconfident persons tend to overestimate the probabilities of favorable events and underestimate their range. On the other hand, underconfident managers tend to overestimate the probability of unfavorable events and overestimate their range.

With respect to optimism, Gibran's (1951) analogy is that optimists see a rose and pessimists see the thorns of the rose. Another analogy often used is that of a half full (optimism) or half empty glass (pessimist). Taylor and Brown (1988) state that optimists believe that the future will be great for them. They also feel more capable, skilled and knowledgeable than their peers.

To build on existing findings and develop a deeper, broader understanding of these biases, there is a need for additional research to: 1. Define overconfidence and optimism biases to distinguish them from one another. 2. Provide this kind of analysis using a probabilistic framework and include an expanded, balanced view of biases of their counterparts – Underconfidence and Pessimism. 3. Examine configurations of related biases to reveal their interplay and holistic views. 4. Express the biases graphically to improve understanding through visualization. 5. Explore the probabilistic properties of the biases and their impact on the moments of the distribution including the mean, the overall and downside risk and value-at-risk.

Accordingly, the next section expands the biases considered and provides their definitions. Section 3 presents the framework including the skewed normal distribution in mathematical and graphical forms to represent a baseline, rational view. Section 4 expresses in probability distributions the individual biases. Section 5 compare the perceptions of overconfident, underconfident and rational managers on expected value and risk characteristics using Monte Carlo simulations. Summary and conclusions are presented in section 6.

2. Definition of Biases

Moore and Healy (2008) describe three main characteristics of overconfident individuals. The first characteristic relates to the overestimation of actual performance due to illusion of control and planning fallacy. Such individuals tend to overestimate their ability of control and underestimate the time needed to complete a task, e.g., Langer (1975). The second characteristic is overplacement, which occurs when overconfident individuals believe that are better than others. An interesting research found that 93% of US drivers believe that are better than average drivers, e.g., Svenson (1981). The final characteristic is over precision. When individuals were asked to estimate the confidence intervals around their answers, the estimated intervals were found to be too narrow. Over the years, the overconfidence bias was explained using mainly conceptual probabilistic statements, such as, “overconfident traders perceive the distribution of portfolio returns to be too tight and underconfident traders too loose”, e.g., Kyle and Wang (1997).

Overconfident managers become more confident when reality is in line with their beliefs, but do not decrease their confidence proportionately, when reality is contradictory. Underconfident managers, on the other hand, may exhibit the opposite asymmetric behavior. Consequently, the probability distribution for overconfident managers, over time, may become more positively skewed with narrower tails than those of rational managers. In contrast, the distributions of underconfident managers may become more negatively skewed with wider tails.

The unrealistic optimism bias characterizes individuals who believe that negative events are less likely to happen to them than to others and vice versa, e.g., Weinstein (1980), Weinstein and Lachendro (1982) and Weinstein and Klein (1996). Consequently, these individuals tend to underestimate the likelihood of negative events and overestimate that of positive events. Managers possessing the bias expect good than bad things to occur in their life, e.g., Kunda (1987). This thinking leads managers to believe that they are invulnerable and have unrealistic positively expectations. Therefore, they underestimate the possibility of failure, e.g., March and Shapira (1987). Pessimists, on the other hand, have the opposite beliefs, e.g., Scheier and Carver (1985).

To understand the various biases, it is useful to compare them to a rational view that serves as a baseline. The rational view is defined as one that represents the actual reality without psychological distortions. In other words, rational individuals are assumed to perfect foresight of the true distribution of economic outcomes.

3. Probabilistic Framework

This section develops a probabilistic framework based on the skewed normal distribution (SN) to model and understand the impact of behavioural biases on managerial perceptions about the expected value, the variance and other risk measures of important decision-making economic variables. Such variables include, among others, the return of a portfolio, a project's cash-flow stream and a company's earnings.

The use of the SN is motivated by its relative simplicity and the fact that is well-known to social scientists, therefore, its skewed extension is easier to grasp and relate to behavioral biases. It is worth noting that the employment of other skewed distributions, such as, the EGB2 of McDonald and Xu (1995) or the skewed generalized t of Theodossiou (1998), complicate the analysis and results and do not provide any extra benefit to the issues investigated in this paper.

The SN is a continuous three-parameter probability distribution. Each of its parameters can be used to capture managerial biases about future outcomes of economic variables. The parameters together can also help to represent and compare various belief configurations: 1. Rational, 2. Overconfident and Optimistic, and 3. Underconfident and Pessimistic.

Skewed Normal Distribution

The values of an economic variable under consideration, denoted by *x*, are modeled as a non-centered SN distribution, defined by

$$dF_x\left(x\right)= f_x dx = \frac{1}{\varphi\sqrt{2\pi}}\exp\left(-\frac{1}{2}\frac{\left(x-m\right)^2}{\left(1+sgn\left(x-m\right)\lambda\right)^2\varphi^2}\right)dx \quad , \tag{1}$$

where m is the mode of x, φ is a tail parameter and λ is an asymmetry parameter defined over the close interval [–1, 1] and sgn is the sign function taking the value of –1 when $x < m$ and the 1, when $x > m$. The mode m is the maximum likelihood point of the distribution, i.e., $dF_x(x = m) \geq dF_x(x)$, for $x \in \mathbb{R}$. The scaling constant φ, which relates to the standard deviation of x, controls the tails and the asymmetry coefficient λ the shape of the distribution. A negative λ yields a distribution skewed to the left and a positive λ a distribution skewed to the right. In the symmetric case $\lambda = 0$.

Probability Decomposition and Moments

Consider the transformation $x(z) = m + (1 + sgn(x - m)\lambda\varphi z$. The probability mass function in (1) can be rewritten equivalently as

$$dF_x\left(x\right) = \left(1 + sgn\left(z\right)\lambda\right) dF_z \tag{2}$$

where $dF_z\left(z\right) = f_z dz = \left(1/\sqrt{2\pi}\right)\exp\left(-z^2/2\right) dz$ is the well-known standard normal distribution and $sgn(x - m) = sgn(z)$. This decomposition plays a vital role in the derivation of the main results.

It follows easily from equation (2) that the probability of $x \leq m$ is

$$p = P\left(x \leq m\right) = \int_{-\infty}^{m} dF_x = \left(1 - \lambda\right)\int_{-\infty}^{0} dF_z = \frac{1 - \lambda}{2}, \tag{3}$$

because $\int_{-\infty}^{0} dF_z = \frac{1}{2}$. On the other hand, the probability $x > m$ is

$$r = P\left(x > m\right) = \int_{m}^{\infty} dF_x = \left(1 + \lambda\right)\int_{0}^{\infty} dF_z = \frac{1 + \lambda}{2} = 1 - p. \tag{4}$$

The latter two probabilities are quite important in explaining the link between the psychological biases and the perceptions of managers about the mean, variance and other moments of economic variables used in decision making. It can be easily confirmed that the asymmetry parameter

$$\lambda = r - p \tag{5}$$

is equal to the difference between the probability mass to the right and left of the mode.

The mean and variance of x, derived in Appendix 1, are respectively

$$\mu = E(x) = m + \sqrt{8/\pi}\lambda\varphi = m + 1.5958\lambda\varphi \tag{6}$$

and

$$\sigma^2 = \text{var}(x) = \left(1 + (3 - 8/\pi)\lambda^2\right)\varphi^2 = \left(1 + 0.4535\lambda^2\right)\varphi^2. \tag{7}$$

Observe that both moments are functions of the tail and asymmetry parameters φ and λ. Moreover, the mean depends on the mode of the distribution.

Downside and Upside Risk

The equations for the downside and upside risk (standard deviation) of x are respectively (see Appendix 1 for the derivations):

$$\sigma_D = \sqrt{1 - 2/\pi}\,(1-\lambda)\varphi = 0.6028\,(1-\lambda)\varphi \tag{8}$$

and

$$\sigma_U = \sqrt{1 - 2/\pi}\,(1+\lambda)\varphi = 0.6028\,(1+\lambda)\varphi. \tag{9}$$

Value-at-Risk and Expected Shortfall

Value-at-risk (VaR) is a measure of the maximum loss when the worse outcome with a small probability q are excluded. The value of q is typically set to 0.01 or 1%. Simply, the $VaR_q = -x_q$, where x_q is the quantile value of x, obtained from the inversion of the following equation

$$F_x\left(x_{q,\lambda}\right) = \int_{-\infty}^{x_{q,\lambda}} dF_x(x) = (1-\lambda)\int_{-\infty}^{z_{q,\lambda}} dF_z(z) = q \;\text{ or }\; \int_{-\infty}^{z_{q,\lambda}} dF_z(z) = \frac{q}{1-\lambda},$$

where $x_{q,\lambda} = m + (1-\lambda)\varphi z_{q,\lambda}$. That is,

$$x_{q,\lambda} = F_x^{-1}(q) \text{ and } z_{q,\lambda} = F_z^{-1}\left(q/(1-\lambda)\right).$$

Note that because $q < p = (1 - \lambda) / 2$, $x_{q,\lambda} < 0$ and $z_{q,\lambda} < 0$. Let $z_q = z_{q,\lambda=0}$ be the quantile value for the rational-expectations manager that satisfies the equation $\int_{-\infty}^{z_q} dF_z(z) = q$. It can be easily confirmed from the above equations that for

$$\lambda < 0, \int_{-\infty}^{z_{q,\lambda}} dF_z(z) < \int_{-\infty}^{z_q} dF_z(z) \text{ and } z_{q,\lambda} < z_q < 0 \text{ or } -z_{q,\lambda} > -z_q > 0$$

and for (10)

$$\lambda > 0, \int_{-\infty}^{z_{q,\lambda}} dF_z(z) > \int_{-\infty}^{z_q} dF_z(z) \text{ and } z_q < z_{q,\lambda} < 0 \text{ or } -z_q > -z_{q,\lambda} > 0.$$

The value-at-risk is

$$VaR_{q,\lambda} = -x_{q,\lambda} = -m - (1-\lambda)\varphi z_{q,\lambda}. \tag{11}$$

The expected shortfall (*ES*) is the expected value of the loss exceeding the value-at-risk,

$$ES_{q,\lambda} = -E\left(x \middle| -x > VaR_{q,\lambda}\right) = -m - (1-\lambda)\varphi E\left(z \middle| -z > -z_{q,\lambda}\right). \tag{12}$$

It follows easily from equations (10) – (1 1) that for

$$\lambda < 0, -x_{q,\lambda} > -x_q \text{ and } ES_{q,\lambda} > ES_q$$

and (13)

$$\lambda > 0, -x_{q,\lambda} < -x_q \text{ and } ES_{q,\lambda} < ES_q,$$

where ES_q is the expected shortfall for the rational manager.

Equations (10) – (13) are quite important in explaining the impact of behavioral biases on the perceptions of managers about value-at-risk measures.

4. Behavioural Biases Implications

The probabilistic framework developed in the previous section is employed to explain the impact of behavioral biases on managers' perceptions about important decision-making variables such as the expected value, the overall risk, downside risk, value-at-risk and expected shortfall. Our approach is in line with the Bayesian approach framework used in Morris (1974) and (1977) and Van den Steen (2001), (2002), (2004) and (2011).

Regardless of behavioural bias, the types of managers examined are assumed to have the same perception of the mode of x. This assumption appears to be reasonable because the mode is the most likely (maximum likelihood) value of the distribution of x. Moreover, it facilitates the comparison of how they perceive the expected value and variance of the outcome relative to the objective or rational manager.

Rational Expectations Manager

The case of a manager with unbiased beliefs or perfect hindsight of the true distribution of x, is used as a benchmark to contrast the differences between managerial biases. This type of manager will be referred as the rational expectations manager. To simplify the analysis and without affecting the main conclusions in the paper, the true distribution of the random variable x of the rational-expectations manager is assumed to be symmetric and normal. That is, $\lambda = 0$, $\varphi = 1$, thus $dF_x = dF_z$ and $z = (x - m) / \varphi$. The expected value and standard deviation of x associated with the rational-expectations manager are respectively $\mu = m$ and $\sigma = \varphi$; see equations (6) and (7).

Overconfidence vs. Underconfidence

Overconfidence is a bias where an individual's subjective judgement of a positive or favorable event is reliably greater and has a narrower range than that of the objective judgement. Overconfident managers tend to overestimate the probability of favorable events and underestimate the probability of unfavorable events. Moreover, the range of their forecasts tends to be narrower. In the management literature, overconfidence has been identified with the overestimation of the actual value, as well as, the precision of a business outcome.

In short, the overconfidence bias involves miscalibration of subjective probabilities and tails of the distribution, e.g., Ben-David et al. (2013). Overconfident managers overestimate managerial perrformance, expected portfolio returns, a project's expected cash flows, etc. Moreover, they tend to overvalue the stock of their firm and consequently incentive options provided to them, e.g., Palmon et al. (2008) and Palmon and Venezia (2013) and (2015).

Underconfident managers, on the other hand, exhibit the opposite behavior. They underestimate the actual mean value of an outcome and attach a wider range of confidence intervals around the mean. Moreover, they tend to underestimate the probablility of favorable events and overestimate that of unfavorable events.

The perceptions of overconfident and underconfident managers can be characterized using the tail and asymmetric parameters φ and λ. The parameters for the overconfident manager are denoted by φ_o and λ_o and the underconfident by φ_u and λ_u. In short, the biases imply that $\varphi_o < \varphi < \varphi_u$ and $\lambda_o < 0 < \lambda_u$.

Proposition 1 An overconfident manager overestimates the probability of favorable outcomes and imposes tighter tails on the probability distributions of decision-making variables. This miscalibration leads to: (a) positively skewed subjective probability distributions for the variables, (b) overestimation of their expected values, (c) underestimation of their downside risk but not necessarily overall risk and (d) underestimation of Value-at-Risk and expected shortfall.

Below we provide the proof of the above proposition.

Note that the assignment of larger probabilities on favorable outcomes implies smaller probabilities for the unfavorable outcomes. Without loss of generality, assume that the values of x

larger than the mode *m* are favorable outcomes. Consequently, smaller values are unfavorable. This miscalibration implies that, for any value $c > 0$,

$$dF_x\left(m-c\middle|m,\varphi_o,\lambda_o\right) < dF_x\left(m+c\middle|;m,\varphi_o,\lambda_o\right),$$

where dF_x is the probability mass function for *x* and φ_o and λ_o are respectively the values of the tail and asymmetry parameters perceived by the overconfident manager. This inequality states that for any symmetric pair of values around the mode *m*, the probability of occurrence of $x = m + c$ is larger than that of $x = m - c$.

The inequality can be re-written as

$$\left(1-\lambda_o\right)dF_z\left(-z\right) < \left(1+\lambda_o\right)dF_z\left(z\right), \text{for } z > 0,$$

where $z = (x - m) / \varphi_o$ is a standardized random variable and dF_z is the standard normal distribution. Because $dF_z(-z) = dF_z(z)$, it implies that the asymmetry parameter for the overconfident manager $\lambda_o > 0$, thus dF_x is positively skewed. Observe that the miscalibration of subjective probabilities by the overconfident managers leads to a positively skewed subjective probability distribution for *x*. This proves part (a) of the proposition.

The expected value of *x* is

$$\mu_o = m + 1.5958\,\varphi_o\lambda_o > m = \mu;$$

see equation (6). Because $\lambda_o > 0$, $\mu_o > \mu$, thus the overconfident manager will overestimate the expected value of *x*; part (b) of the proposition.

The standard deviation of *x*, see equation (7), is

$$\sigma_o = \sqrt{1+0.4535\lambda_o^2}\,\varphi_o.$$

Although theory postulates that $\varphi_o < \varphi = \sigma$, this does not necessarily imply that an overconfident manager will underestimate risk or that $\sigma_o < \sigma$. That is, σ_o will be smaller than σ, provided that

$$\sigma_o = \sqrt{1 + 0.4535\,\lambda_o^2}\,\varphi_o < \sigma = \varphi$$

or

$$\varphi_o < \varphi \Big/ \sqrt{1 + 0.4535\,\lambda_o^2}\,.$$

In the opposite case, the manager will overestimate risk.

The downside risk perceived by an overconfident manager, given by equation (8), is

$$\sigma_{o,D} = \sqrt{1 - 2/\pi}\,(1 - \lambda_o)\,\varphi_o = 0.6028\,(1 - \lambda_0)\,\varphi_o\,.$$

Because $\varphi_o < \varphi$ and $\lambda_o > 0$,

$$\sigma_{o,D} = 0.6028\,(1 - \lambda_0)\,\varphi_o < 0.6028\,(1 - \lambda_0)\,\varphi < 0.6028\,\varphi = \sigma_D\,,$$

where $\sigma_D = 0.6028\,\varphi$ measures downside risk for the rational agent. The above result is the proof for part (c) of the proposition.

Because $\lambda_o > 0$, it follows from equation (10) that $-z_q > -z_{q,o} > 0$, where $z_{q,o}$ and z_q are respectively the quantile values for the overconfident and rational managers. Because $\varphi_o < \varphi$ and $\lambda_o > 0$, the overconfidence manager's VaR is

$$VaR_{q,o} = -x_{q,o} = -m - (1 - \lambda_o)\,\varphi_o z_{q,o} < -m - (1 - \lambda_o)\,\varphi_o z_q < -m - \varphi z_q = -x_q = VaR_q\,,$$

where VaR_q is the rational manager's VaR. Moreover, it follows from (13) that for

$$\lambda_o > 0,\ ES_{q,\lambda} < ES_{q,},$$

where ES_q is the rational manager's expected shortfall. These prove part (d) of the proposition. ■

Figure 1 provides a graphical illustration of the probability distributions of the rational and overconfident managers. For the rational manager, the three parameters are set to $m = 0$, $\varphi = 1$ and

$\lambda = 0$. The distribution is represented by the dotted curve. For the overconfident manager the parameters are set to $m = 0$, $\varphi = 0.8$ and $\lambda = 0.4$. Observe that the distribution is skewed to the right. For these parameters, the overconfident manager attaches a 0.7 probability for the values on the right of the mode vs. 0.3 probability for values on the left of the mode. The expected value of x is 0.51 and the standard deviation is 0.83, thus the overconfident manager overestimates the expected value and underestimate the risk of x or forms lower confidence intervals around the mean.

<u>**Proposition 2**</u> **An underconfident manager underestimates the probability of favorable outcomes and imposes wider tails on the probability distributions of decision-making variables. This miscalibration leads to: (a) negatively skewed subjective probability distributions for the variables, (b) underestimation of their expected values and (c) overestimation of their overall risk, downside risk, value-at-risk and expected shortfall.**

The proof for parts (a) and (b) of proposition 2 is like that of proposition 1, therefore, is omitted. Part (b) implies that the asymmetry parameter $\lambda_u < 0$. The overall risk of x associated with the underconfident manager is

$$\sigma_u = \sqrt{1 + 0.4535\,\lambda_u^2}\,\varphi_u > \sigma = \varphi\,,$$

because $\varphi_u > \varphi = \sigma$ and $\sqrt{1 + 0.4535\,\lambda_u^2} > 1$. Similarly, because $\lambda_u < 0$ and $\varphi_u > \varphi$, downside risk is

$$\sigma_{u,D} = 0.6028\left(1 - \lambda_u\right)\varphi_u > 0.6028\,\varphi_u > 0.6028\,\varphi = \sigma_D\,,$$

where $\sigma_D = 0.6028\,\varphi$ measures downside risk for the rational agent. Because $\lambda_u < 0$, equation (10) implies that $-z_{q,u} > -z_q > 0$, where $z_{q,u}$ and z_q are respectively the quantile values for the underconfident and rational managers. Because $\varphi_u > \varphi$, the underconfidence manager's VaR is

$$VaR_{q,u} = -x_{q,u} = -m - \left(1 - \lambda_u\right)\varphi_u z_{q,u} > -m - \left(1 - \lambda_u\right)\varphi_u z_q > -m - \varphi z_q = -x_q = VaR_q\,,$$

where VaR_q is the rational manager's VaR measure. Moreover, it follows from equation (13) that for $ES_{q,\lambda} > ES_{q,\lambda=0}$. These prove part (c) of the proposition. ■

Figure 2 presents the graphical illustration of the probability distributions of the rational and the underconfident manager. For the rational manager, the three parameters are set to $m = 0$, $\varphi = 1$ and $\lambda = 0$ and for the underconfident manager to $m = 0$, $\varphi = 1.2$ and $\lambda = -0.4$. Observe that the distribution is skewed to the left. For these parameters, the underconfident manager attaches a 0.3 probability for the values on the right of the mode and 0.7 probability for values on the left of the mode. The expected value of x is –0.77 and the standard deviation is 1.24, thus the underconfident manager underestimates the expected value and overestimates the risk of x.

Optimist vs. Pessimist

Weinstein (1980) studied the tendency of people to be unrealistically optimistic about future life events. He found that cognitive and motivational considerations influence the amount of optimistic bias of different events. Optimism can be defined in different interrelated ways. In general, optimistic people believe that undesirable (desirable) events are less (more) likely to happen to them than to others. In other words, optimistic people tend to overestimate the probability of events perceived to be positive and underestimate that of events perceived to be negative.

Proposition 3 Managers characterized by optimism underestimate the probability of unfavorable events and overestimate the probability of favourable events. This miscalibration leads to: (a) positively skewed subjective probability distributions for the variables, (b) overestimation of their expected values, (c) overestimation of overall risk and (d) underestimation of their downside risk, value-at-risk and expected shortfall.

The proof for parts (a) and (b) of the proposition is like that of proposition 1, therefore, is omitted. Part (b) implies that the asymmetry parameter $\lambda_{op} > 0$. The overall risk of x associated with the optimistic manager (definition does not say anything about the tails) is

$$\sigma_{op} = \sqrt{1+0.4535\,\lambda_{op}^2}\,\varphi > \sigma = \varphi\,,$$

because $\sqrt{1+0.4535\,\lambda_{op}^2} > 1$. Similarly, because $\lambda_{op} > 0$, downside risk is

$$\sigma_{op,D} = 0.6028\left(1-\lambda_{op}\right)\varphi < 0.6028\,\varphi = \sigma_D\,,$$

where $\sigma_D = 0.6028\,\varphi$ is downside risk for the rational manager. Because $\lambda_{op} > 0$, equation (10) implies that $-z_q > -z_{q,op} > 0$, where $z_{q,op}$ and z_q are respectively the quantile values for the optimistic and rational managers. Because $\lambda_{op} > 0$, the optimistic manager's VaR is

$$VaR_{q,op} = -x_{q,op} = -m-\left(1-\lambda_{op}\right)\varphi z_{q,op} < -m-\varphi z_q = -x_q = VaR_q\,,$$

where VaR_q is as defined previously. Moreover, it follows from equation (13) that for $\lambda_{op} > 0$, $ES_{q,\lambda} < ES_q$. This proves part (d) of the proposition. ■

<u>**Proposition 4**</u> **Managers characterized by pessimism overestimate the probability of unfavorable events and underestimate the probability favourable events. This miscalibration leads to: (a) negatively skewed subjective probability distributions for the variables under consideration, (b) underestimation of their expected values and (c) overestimation of their overall risk, downside risk, value-at-risk and expected shortfall.**

The proof for parts (a) and (b) is like that of proposition 1, therefore, is omitted. Part (b) implies that $\lambda_{pe} < 0$. The overall risk of x associated with the pessimistic manager is

$$\sigma_{pe} = \sqrt{1+0.4535\,\lambda_{pe}^2}\,\varphi > \sigma = \varphi\,,$$

because $\sqrt{1+0.4535\lambda_{pe}^2} > 1$. Similarly, because $\lambda_{pe} < 0$, downside risk is

$$\sigma_{pe,D} = 0.6028\left(1-\lambda_{pe}\right)\varphi > 0.6028\,\varphi = \sigma_D,$$

where $\sigma_D = 0.6028\,\varphi$ is the rational manager's downside risk. Because $\lambda_{pe} < 0$, it follows from equation (1) that $-z_{q,pe} > -z_q > 0$, where $z_{q,pe}$ and z_q are respectively the quantile values for the pessimist and rational managers. Moreover, the pessimist's VaR and expected shortfall are

$$VaR_{q,pe} = -x_{q,pe} = -m-\left(1-\lambda_{pe}\right)\varphi z_{q,pe} > -m-\left(1-\lambda_{pe}\right)\varphi z_q > -m-\varphi z_q = -x_q = VaR_q$$

and

$$ES_{q,\lambda} > ES_q. \blacksquare$$

Overconfidence vs. Optimist

Below we compare the forecasting characteristics of overconfident and optimistic managers

Proposition 5. Optimistic managers facing the same mode and asymmetry parameter as overconfident managers, overestimate to a greater extent the mean and overall risk and underestimate to a lesser extent downside risk, value-at-risk and expected shortfall of decision-making variables.

Under the assumptions that optimistic and overconfident managers face the same mode and asymmetry parameters ($\lambda_{op} = \lambda_o$) and because $\varphi_o < \varphi$, the following results can be easily obtained

$$\mu_{op} = m + 1.5958\varphi\,\lambda_{op} > \mu_o = m + 1.5958\varphi_o\lambda_o,$$

$$\sigma_{op} = \sqrt{1+0.4535\lambda_{op}^2}\,\varphi > \sigma_o = \sqrt{1+0.4535\lambda_o^2}\,\varphi_o,$$

$$\sigma_{op,D} = \sqrt{1-2/\pi}\left(1-\lambda_{op}\right)\varphi > \sigma_{o,D} = \sqrt{1-2/\pi}\left(1-\lambda_o\right)\varphi_o,$$

$$VaR_{q,o} = -m-\left(1-\lambda_o\right)\varphi_o z_{q,o} < VaR_{q,op} = -m-\left(1-\lambda_{op}\right)\varphi z_{q,o} < VaR_q = -m-\varphi z_q$$

and

$$ES_o < ES_{q,op}.$$

Note that because $\lambda_{op} = \lambda_o$, $z_{q,o} = z_{q,op}$. ■

Figure 3 shows the configurations of biases and the differences between the overconfident, the optimistic and the rational managers. The curves for the overconfident and rational managers are those of figures 1. Note that the tails of the distribution for the optimistic manager are wider and the probability mass around the mode smaller. Consistent with the results of proposition 3, the optimistic manager overestimates the expected value and overall risk. Note that the overestimation bias is greater than that of the overconfident manager, i.e., $\mu_{op} = 0.64$ vs. $\mu_o = 0.51$.

The visualization of biases made possible through representation of statistical distributions shows that the optimism bias is a special case of the overconfidence bias. The distributions in figures help to visualize the individual biases and their interplay within a configuration.

5. Simulations

The mean and risk perceptions of managers characterized by overconfidence, underconfidence, optimism and pessimism are compared to those of rational managers using monte Carlo simulations. The comparisons are illustrated by means of an investment portfolio and a capital budgeting example. For the analysis, rational managers are assumed to have perfect foresight of the true distribution of the decision variables under consideration, i.e., the return of the portfolio or the growth rates of cash-flows of the capital project.

Without loss of generality, the mode of the distribution of portfolio returns or cash-flow growth rates is taken to be zero. Moreover, the values of the tail parameter φ in the cases of rational, optimist and pessimist managers are set to 0.1 (or 10%) and the asymmetry parameter λ to zero. These parameters for an overconfident manager are set to $\lambda = 0.4$ and $\varphi = 0.08$ and an underconfident manager to $\lambda = –0.4$ and $\varphi = 0.12$.

Investment Portfolio

The end-of-period portfolio value is $V_1 = V_0\ (1 + x)$, where V_0 is the initial portfolio value and x is a normally distributed return with mean zero ($\mu = m = 0$) and standard deviation 0.1 ($\sigma = 0.1$). Table 2 presents the expected value, $E(V_1 \mid C)$, overall risk, $\sigma(V_1 \mid C)$, the downside risk $\sigma_D(V_1 \mid C)$, and the 1% value-at risk, $\text{VaR}_{1\%}$ and expected shortfall, $ES_{1\%}$, for the five behavioural configurations, denoted by C. These values are based on a monte Carlo sample of 100,000 randomly generated portfolio returns.

Specifically, the expected value, overall risk and downside risk for each configuration are computed using respectively the arithmetic mean, the standard deviation of all portfolio values and the standard deviation of portfolio values less than the initial value of $V_0 = 100$. The $VaR_{1\%}$ is quantile value of each simulated sample at the 1% and the $ES_{1\%}$ is the arithmetic mean of portfolio values less than the value-at-risk.

The results depict that the overconfident and the optimistic managers overestimate the end-of-period expected portfolio value. The extent of overestimation bias is larger in the case of optimistic managers. Underconfident and pessimistic managers, on the other hand, underestimate expected portfolio values and the underestimation bias is larger for underconfident managers.

The results also depict, that overconfident managers underestimate overall, downside risk, value-at-risk and expected shortfall values. The downside, the value-at-risk and the expected shortfall measures are also underestimated by optimists and overestimated by those characterized by underconfidence and pessimism. The overestimation, is however, more pronounced for the underconfident managers.

Capital Budgeting

For the capital budgeting example, let project's cashflows be CF_0 = –111.29 and CF_t = 25 (1 + x), for t = 1, 2, …,5, where x is as defined previously. For simplicity, the cost of capital is assumed to be non-stochastic and equal to k = 0,04. The random behaviour of the project's net present value

$$NPV = CF_0 + \sum_{t=1}^{N} CF_t \left(1+k\right)^{-t} = -111.29 + \sum_{t=1}^{5} 25\left(1+x\right)\left(1+0.04\right)^{-t},$$

is driven by the cashflows' growth rate x. Table 3 presents the mean, the standard deviation, the downside standard deviation and the 1% VaR and ES measures of the project's NPV based on a monte Carlo simulation of 100.000 randomly generated values for each configuration. Observe that the conclusions are identical to those of Table 2, therefore, the discussion is omitted.

6. Summary and Conclusions

This paper develops a probabilistic framework based on the skewed normal distribution (SN) to model the managerial biases of overconfidence and unrealistic optimism and their counterparts of underconfidence and pessimism. The framework is used as a tool to compare the differences and similarities of these biases and analyze their impact on the expected value, overall risk, downside risk, value-at-risk and expected shortfall of decision-making variables, such as the return of an investment, the cash-flow of a capital project and the earnings of a company.

Theory postulates that an overconfident manager overestimates the probability of favorable outcomes and imposes tighter tails on the probability distributions of economic variables used in decision making. This miscalibration leads to a positively skewed subjective probability distributions for the variables under consideration, and overestimation of their expected values and an underestimation of their overall risk, downside risk, value-at-risk and expected shortfall. An underconfident manager underestimates the probability of favourable outcomes and the miscalibration leads to a negatively skewed distribution, an underestimation of their expected values and overestimation of overall risk, downside risk, value-at-risk and expected shortfall.

Managers characterized by optimism underestimate the probability of unfavorable events and overestimate the probability favourable events. Nothing, however, is stated about how optimists view the tails of the perceived probability distribution. Like in the case of the overconfident managers, this miscalibration leads to a positively skewed subjective probability distributions for the variables under consideration, an overestimation of their expected values and an underestimation of their downside risk. However, the optimist will overestimate the overall risk of the variables. Compared to the overconfident managers, their over estimation bias of the mean will be larger, but the underestimation biases of downside risk, value-at-risk and expected shortfall will be smaller.

The implications of these biases are further investigated using Monte Carlo simulations of a portfolio and a capital budgeting example. In all cases, the simulations confirm the analytical derived using the probabilistic framework developed in the paper.

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Appendix 1. Moment Function - Skewed Normal Distribution

Based on equation (2), the moment function of *x* in excess of the mode, is

$$M_s = E(x-m)^s = \int_{-\infty}^{\infty} (x-m)^s \, dF_x = \varphi^s (1-\lambda)^{s+1} \int_{-\infty}^{0} z^s dF_z + \varphi^s (1+\lambda)^{s+1} \int_{0}^{\infty} z^s dF_z \; .$$

Because dF_z is symmetric around zero, $\int_{-\infty}^{0} z^s dF_z = (-1)^s \int_{0}^{\infty} z^s dF_z$, thus

$$M_s = \varphi^s \left[(-1)^s (1-\lambda)^{s+1} + (1+\lambda)^{s+1} \right] \int_{0}^{\infty} z^s dF_z \; .$$

Let $t = z^2/2$, thus $z = 2^{\frac{1}{2}} t^{\frac{1}{2}}$, $dz = 2^{-\frac{1}{2}} t^{-\frac{1}{2}} dt$, $z^s dz = 2^{\frac{s}{2}-\frac{1}{2}} t^{\frac{s+1}{2}-1} dt$ and

$$\int_{0}^{\infty} z^s dF_z = \frac{1}{\sqrt{2\pi}} \int_{0}^{\infty} z^s e^{-z^2/2} dz = \frac{1}{\sqrt{2\pi}} 2^{\frac{s}{2}-\frac{1}{2}} \int_{0}^{\infty} t^{\frac{s+1}{2}-1} e^{-t} dt = \frac{1}{\sqrt{\pi}} 2^{\frac{s}{2}-1} \Gamma\left(\frac{s+1}{2}\right).$$

The substitution of this result into M_s gives

$$M_s = \frac{1}{\sqrt{\pi}} 2^{\frac{s}{2}-1} \left[(-1)^s (1-\lambda)^{s+1} + (1+\lambda)^{s+1} \right] \Gamma\left(\frac{s+1}{2}\right) \varphi^s . \qquad \text{(A1)}$$

For $s = 1$, $M_1 = E(x-m) = \frac{1}{\sqrt{\pi}} 2^{-\frac{1}{2}} \left[(-1)^1 (1-\lambda)^2 + (1+\lambda)^2 \right] \Gamma(1) \varphi = 2\sqrt{2/\pi}\,\lambda\varphi$, thus

$$\mu = E(x) = m + 2\sqrt{2/\pi}\,\lambda\varphi = m + 1.5958\lambda\varphi . \qquad \text{(A2)}$$

For $s = 2$, $M_2 = \frac{1}{\sqrt{\pi}} 2^{\frac{2}{2}-1} \left[(-1)^s (1-\lambda)^3 + (1+\lambda)^3 \right] \Gamma\left(\frac{3}{2}\right) \varphi^2 = \left(1+3\lambda^2\right)\varphi^2$, thus

$$\sigma^2 = \mathrm{var}(x) = M_2 - M_1^2 = \left(1 + (3 - 8/\pi)\lambda^2\right)\varphi^2 = \left(1 + 0.4535\,\lambda^2\right)\varphi^2 . \qquad \text{(A3)}$$

Downside Risk

In general, the downside moment function of *x* in excess of the mode is

$$M_s^- \equiv E\left((x-m)^s \middle| x \le m \right) = \frac{1}{P(x \le m)} \int_{-\infty}^{m} (x-m)^s \, dF_x$$

$$=(-1)^s(1-\lambda)^s\varphi^s 2\int_0^\infty z^s dF_z \ =(-1)^s(1-\lambda)^s\frac{1}{\sqrt{\pi}}2^{\frac{s}{2}}\Gamma\left(\frac{s+1}{2}\right)\varphi^s .$$

For s =1 and 2

$$M_1^- = -(1-\lambda)\frac{1}{\sqrt{\pi}}2^{\frac{1}{2}}\Gamma(1)\varphi = -(1-\lambda)\sqrt{2/\pi}\varphi ,$$

$$M_2^- = \frac{1}{\sqrt{\pi}}2(1-\lambda)^2\Gamma\left(\frac{3}{2}\right)\varphi^2 = (1-\lambda)^2\varphi^2$$

and the downside variance of x is

$$\sigma^2_{x|x\le m} = \mathrm{var}(x|x\le m) = M_2^- - (M_1^-)^2 = (1-\lambda)^2\varphi^2 - (1-\lambda)^2\frac{2}{\pi}\varphi^2 \ .$$

$$=(1-\lambda)^2(1-2/\pi)\varphi^2 = 0.36338\cdot(1-\lambda)^2\varphi^2 . \qquad \text{(A4)}$$

Upside Risk

In general, the downside moment function of x in excess of the mode is

$$M_s^+ \equiv E\left((x-m)^s\middle| x>m\right) = \frac{1}{P(x>m)}\int_m^\infty (x-m)^s dF_x$$

$$=(1+\lambda)^s\varphi^s 2\int_0^\infty z^s dF_z \ =(1+\lambda)^s\frac{1}{\sqrt{\pi}}2^{\frac{s}{2}}\Gamma\left(\frac{s+1}{2}\right)\varphi^s .$$

For s =1 and 2,

$$M_1^+ = E(x-m|x>m) = (1+\lambda)\frac{1}{\sqrt{\pi}}2^{\frac{1}{2}}\Gamma(1)\varphi = (1+\lambda)\sqrt{2/\pi}\varphi \text{, thus}$$

$$M_2^+ = \frac{1}{\sqrt{\pi}}2(1+\lambda)^2\Gamma\left(\frac{3}{2}\right)\varphi^2 = (1+\lambda)^2\varphi^2$$

and the upside variance of x is

$$\sigma^2_{x|x>m} = \mathrm{var}(x|x>m) = M_2^+ - (M_1^+)^2 = (1+\lambda)^2\varphi^2 - (1+\lambda)^2\frac{2}{\pi}\varphi^2 \ .$$

$$=(1+\lambda)^2(1-2/\pi)\varphi^2 = 0.36338\cdot(1+\lambda)^2\varphi^2 . \qquad \text{(A5)}$$

Table 1. Notation of Various Biases Used

Configuration	Notation
Overconfidence	*o*
Underconfidence	*u*
Optimism	*op*
Pessimism	*pe*

Table 2. Managerial Perceptions of Portfolio Expected Value, Overall Risk, Downside Risk and 1% *VaR* and *ES* under the Various Bias Configurations

Configuration	φ	λ	$E(V_1 \vert C)$	$\sigma(V_1 \vert C)$	$\sigma_D(V_1 \vert C)$	$VaR_{1\%}$	$ES_{1\%}$
Rational	0.1	0.0	100	10.00	6.04	23.29	26.78
Overconfident	0.08	0.4	105.11	8.30	2.92	10.29	11.96
Underconfident	0.12	–0.4	92.35	12.38	10.10	41.00	46.59
Optimist	0.1	0.4	106.38	10.35	3.61	12.82	14.93
Pessimist	0.1	–0.4	93.63	10.37	8.40	34.02	38.60

Notes: The above measures are based on one-hundred thousand skewed normal returns for each configuration, generated using the parameter values of φ and λ, given by the second and third columns, respectively. Investment values are computed using the equation $V_1 = 100\ (1 + x)$, where x is the randomly generated return. The values under the columns for $E(V_1 \vert C)$ and $\sigma(V_1 \vert C)$ give respectively the sample mean and standard deviation of the simulated portfolio values of each configuration. The values under the column $\sigma_D(V_1 \vert C)$ give the sample standard deviation of the portfolio values below 100 for each configuration. The last two columns give the 1% value at risk (*VaR*) and expected shortfall (*ES*) measures.

Table 3. Managerial Perceptions of *NPV* Expected Value, Overall Risk, Downside Risk and 1% *VaR* and *ES* Under the Various Bias Configurations

Configuration	φ	λ	$E(NPV \mid C)$	$\sigma(NPV \mid C)$	$\sigma_D(NPV \mid C)$	$VaR_{1\%}$	$ES_{1\%}$
Rational	0.1	0.0	–0.02	4.98	3.00	11.63	13.28
Overconfident	0.08	0.4	5.67	4.11	1.30	3.02	4.09
Underconfident	0.12	–0.4	–8.56	6.21	5.55	24.08	26.66
Optimist	0.1	0.4	7.13	5.17	1.65	3.85	5.21
Pessimist	0.1	–0.4	–7.11	5.17	4.62	20.09	22.14

Notes: The above measures are based on one-hundred thousand skewed normal cash-flow growth rates for each configuration, generated using the parameter values of φ and λ, given by the second and third columns of the table, respectively. Net present values are computed using the equation $NPV = -111.29 + \sum_{t=1}^{5} 25(1+x)(1.04)^{-t}$, where x is the randomly generated cash flow growth rate. The values under the columns for $E(NPV \mid C)$ and $\sigma(NPV \mid C)$ give respectively the sample mean and standard deviation of the simulated *NPV* values of each configuration. The values under the column $\sigma_D(NPV \mid C)$ give the sample standard deviation of the negative *NPV* values for each configuration. The last column gives the 1% value-at-risk measure (*VaR*) and expected shortfall (*ES*) measures.

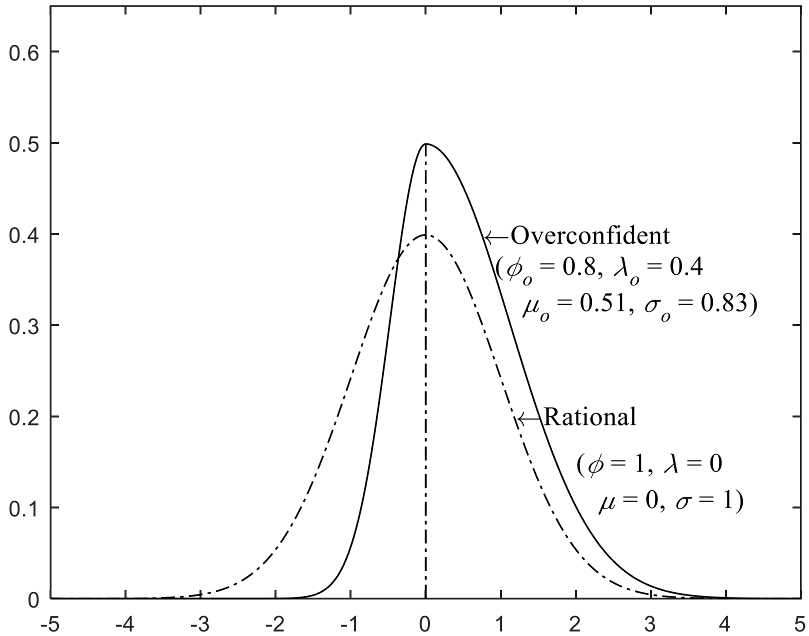

Figure 1. Overconfident model superimposed over the rational model

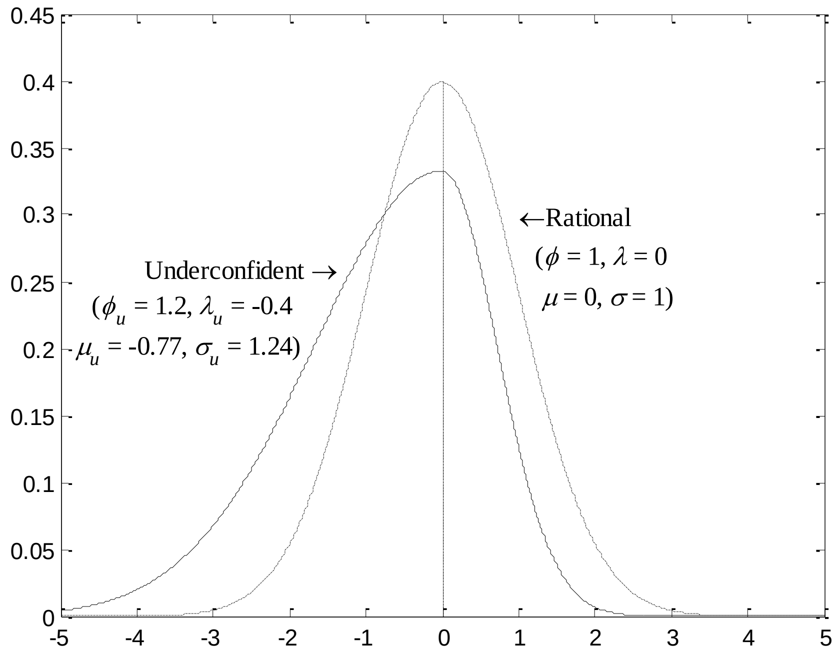

Figure 2. Underconfident model superimposed over the rational model

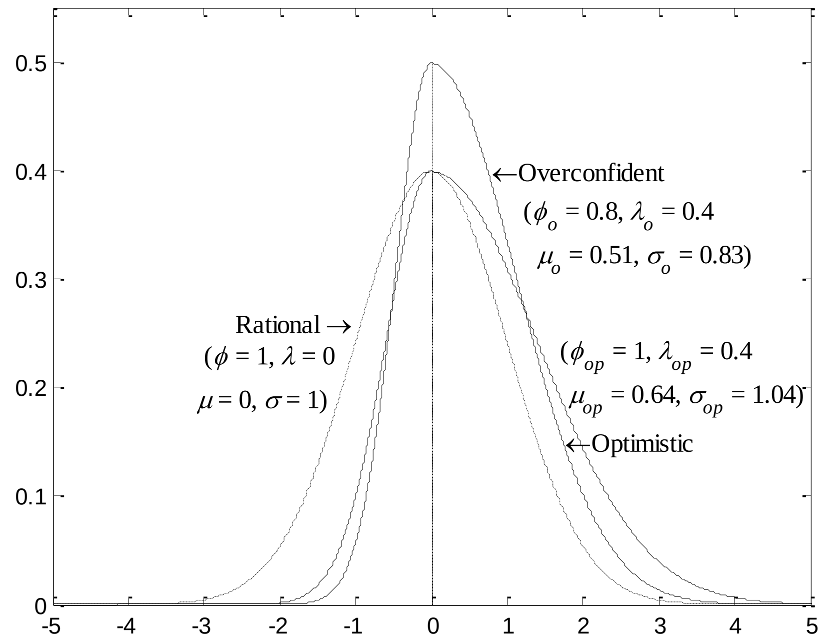

Figure 3. Overconfidence, Rational and Unrealistic Optimism biases configuration model